Volume XXVII. *July, 1908.* *Number 1.*

THE

PHYSICAL REVIEW.

FURTHER MEASUREMENTS OF THE COEFFICIENT OF LINEAR EXPANSION AT LOW TEMPERATURES.

By Herbert G. Dorsey.

IN the previous paper[1] were given data and curves for twelve different substances. In continuing the work a slight modification of the light arrangement has been made, which is more satisfactory and less liable to get out of adjustment. With the exception of the mercury vapor lamp the entire light apparatus is carried by an optical bench and the path of the light rays is entirely in a vertical plane.

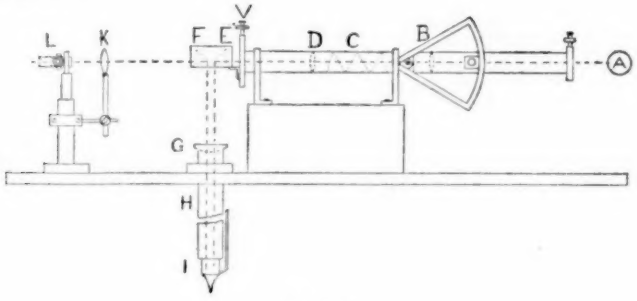

Fig. 1.

Starting from the source *A*, Fig. 1, the light passes through a short, narrow, horizontal slit into the collimator of an old spectrophotometer and is rendered parallel by the lens *B*. It then passes through the train of prisms *C* and is focused by the lens *D* upon the slit *V*. The collimator being hinged at the end *B* it is only

[1] Phys Rev., Vol. XXV., August, 1907.

necessary to raise or lower it to get the desired spectral band at *V*. The green line is used and the same Cooper-Hewitt lamp has now been used for three years, and although the tube is becoming slightly blackened, the light seems as pure as at first and is entirely satisfactory. Some time was devoted to a cadmium lamp, but so far it has seemed advisable to adhere to the mercury vapor arc.

After leaving the slit *V* the green light is reflected vertically downward by the right-angle prism *E* and becomes parallel by the lens *G*. Thence down the tube *H* to the box *I*, up again to the right-angle prism *F*, where it is reflected horizontally through the lens *K* to the micrometer eye-piece *L*. The most difficult part of the adjustment is after the light enters the tube *H*. From the theory of interference in thick plates, if the two plates are parallel, the fringes are produced at infinity. And although the plates, *i. e.*, the ends of the specimen, are seldom parallel, the fringes are still far away. At the same time they must be brought to a focus at the cross wires of the eye piece, as must also the bottom side of the plate on top of the specimen. The lens *G* produces an image of the fringes near its principal focus at the left of the prism *F*, while the image of the plate is formed quite a distance to the left of *L*. By interposing the lens *K*, a conjugate focus of the fringes is formed and the image of the plate is brought nearer. It requires considerable time, however, to get the two images to properly coincide.

It might be of interest to note the method of final adjustment of the specimen. The three projections are first ground down on fine emery paper until they are of nearly equal length as shown by micrometer calipers. Then the steel box is removed from the tube and a plate of black glass on a leveling table is placed under the tube *H*. The specimen is placed upon the black glass and the top wedge of glass placed upon the specimen. Usually interference bands will be seen but seldom of a suitable width. The top wedge is then gently pressed down with the fingers, over one projection or another, until the fringes become wider. This indicates that the specimen is compressed along that projection and that it must be made shorter. It is then removed and, depending upon the specimen, is rubbed slightly on fine emery paper or ordinary paper to remove a slight amount of the material, and replaced upon the black

glass. A few trials generally suffice to produce fringes of a suitable width. In the case of lead pressure alone was sufficient to compress it to the desired length.

This arrangement of apparatus gives an admirable method of testing plates of optical glass to see if the sides are parallel, as it is only necessary to hold it under the tube *H* on a levelling table. Plates up to two centimeters thickness or more can be easily tested and any irregularities detected by the curvature of the fringes. Although many plates thus tested have shown plane sides none have shown parallel sides. During one series of measurements of zinc, parallelism was produced by unequal contraction of the specimen and the plate appeared alternately bright and dark.

The method of keeping the window of the box dry by a current of air from the evaporating liquid has been abandoned for a less elegant but more effective method. It appears that ice and carbon dioxide evaporate even at liquid air temperatures and when the current of air from the Dewar bulb is blown on the box window the moisture and carbon dioxide are frozen on the cold surface. The fact that the carbon dioxide was present was shown many times by the window clearing off as the box was warmed a little past about — 80° C. A small ring-shaped cup is filled with phosphorus pentoxide and placed upon the glass window and left there during the test. It absorbs nearly all the moisture in about half an hour and although disagreeable to work with is more effective than the air.

The following tables give on the absolute temperature scale the middle point of the temperature interval, that at which the coefficient is plotted, the number of fringes counted for the interval and their average, the calculated coefficients, average coefficient between 293° and 93° and description of material.

Discussion of Results.

Perhaps the most interesting curve is that of the first gold specimen. The irregularities in it were quite unexpected and for that reason no. 2 of great purity from the mint was tested. Every precaution was taken to keep it pure. It was at first tested at 10° intervals with the expectation of finding the same irregularities as in no. 1. As they did not appear it was tested at 20° intervals. Find-

ing a smooth curve for the pure gold, no. 3 of doubtful purity was tested and although its coefficient is a trifle higher, still it gave a smooth curve. Another specimen was then made by alloying some of the mint gold with one per cent. platinum and a single test at 10° intervals for the first 100° was made with the same result of a curve smooth within the limits of error of single observations. The average coefficients of the four specimens differ among themselves by less than one per cent. The only explanation offered for the irregu-

GOLD. NO. 2.

From U. S. Philadelphia Mint. 99.998 *Per Cent. Gold,* 0.002 *Per Cent. Silver Specimen Melted and Cooled in a Mold of Graphite. Length* = 1.3523 *cm. Average Width Fringes* = 250 *Divisions of Eye Piece Micrometer. Density* = 19.56. *Average Coefficient between* 293° *and* 93° = 1,335 × 10^{-8}.

Temp.	283°	263°	243°	223°	203°	183°	163°	143°
	14.13	14.22	13.98	13.97	13.60	13.28	13.04	12.68
	13.95	13.98	14.05	13.88	13.78	13.20	12.79	12.85
	14.27	14.21	14.05			13.31		12.50
	14.23	14.17	14.01					
	14.14	14.07						
	14.20							
	14.24							
Average	14.17	14.13	14.02	13.92	13.69	13.26	12.91	12.68
$e \times 10^{-8}$	1,429	1,425	1,415	1,404	1,380	1,339	1,303	1,279
Temp.	123°	103°	288°	278°	268°	258°	248°	238°
	12.00	11.62	7.25	6.98	7.00	7.17	7.02	7.03
	11.95	11.57	7.01	7.13	7.01	7.06	7.00	7.01
			7.15	7.05				
			7.17	7.07				
Average	11.98	11.60	7.15	7.06	7.01	7.11	7.01	7.02
$e \times 10^{-8}$	1,208	1,170	1,440	1,426	1,416	1,435	1,416	1,418

larities of no. 1 is that it probably contained some slight unknown impurity. It was made of gold partly from old jewelry, and part from C. S. Platt Co., gold refiners, New York. It was melted in a crucible and cast in a hot cast iron mold. The specimen was brittle and broke under a blow of the hammer. I have since alloyed some of the pure gold with a small amount of iron but it was not at all brittle. After no. 1 was tested it was alloyed for jewelry and only

a small piece left for analysis. Its curve is shown dotted for comparison with that for the pure gold.

Considerable time was devoted to zinc. Two specimens were tested. The first was melted in its own mold, the second melted and cast. Both were irregular, the different parts expanding at different rates as shown by the fringes rotating and changing width. Furthermore it would not repeat itself. Fringes in one direction

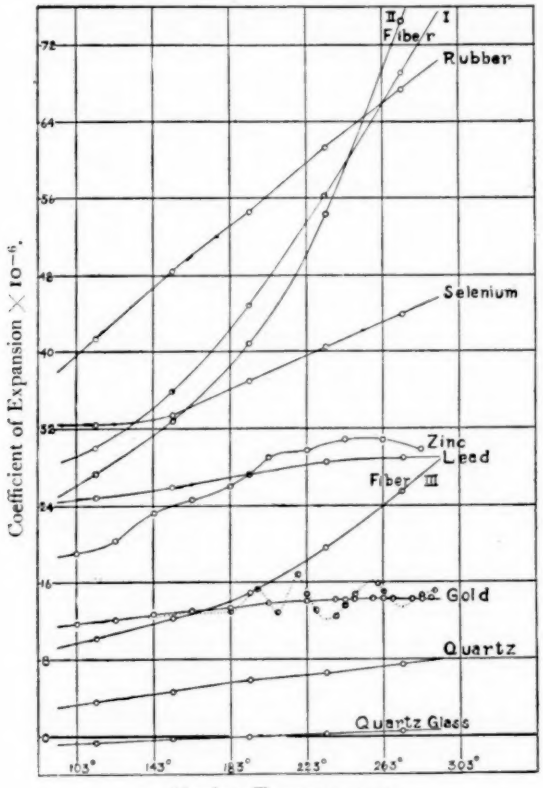

Fig. 2.

would not always equal those in opposite direction for the same interval. The table gives the complete data for the second specimen.

The three curves for fiber are given in hopes they may prove useful to any who are working with this substance at low temperatures. Considering its high expansibility each specimen was remarkably uniform as shown by little rotation of the fringes. Specimens

1 and 2 were cut from different sheets of fiber, both perpendicular to the plane of the sheet. No. 1 was cooled to −180° C. twice and each time the length became less showing a sort of ageing process. Thus the number of fringes when cooled down the first time was 462, while in warming back to same temperature the number was 447. The next time down it was 431.6 while up it was 429.6 mak-

GOLD. NO. 3.

Old Gold Purified by Solution Method Which Would Leave in Any Noble Metals and Possibly Traces of Copper and Silver. Surface of Specimen was Decidedly Crystalline. Length = 0.8856 *cm. Average Width Fringes* = 375. *Density* = 19.15. *Average Coefficient* = 1,346 × 10−8.

Temp.	283°	263°	243°	223°	203°	183°	163°	143°
	9.39	9.47	9.34	9.33	8.90	8.59	8.46	8.10
	9.56	9.40	9.24	9.21	8.71	8.55	8.53	8.18
					8.91			
					8.94			
Average	9.48	9.44	9.29	9.27	8.87	8.57	8.50	8.14
$e \times 10^{-8}$	1,462	1,455	1,431	1,429	1,367	1,321	1,311	1,254
Temp.	123°	103°	288°	278°	268°	258°	248°	238°
	8.15	7.66	4.73	4.66	4.75	4.72	4.56	4.78
	8.05	7.63	4.74	4.82	4.58	4.82	4.60	4.64
Average	8.10	7.65	4.74	4.74	4.72	4.77	4.58	4.71
$e \times 10^{-8}$	1,249	1,180	1,460	1,460	1,454	1,470	1,412	1,451

GOLD. NO. 4.

99 *Per Cent. of the Mint Gold,* 1 *Per Cent. Platinum. Alloy First Melted on Charcoal and then Melted in Graphite Mold. Length* = 0.6304 *cm. Average Width Fringes* = 250. *Average Coefficient* = 1,332 × 10−8.

Temp.	288°	278°	268°	258°	248°	238°
	3.48	3.29	3.42	3.29	3.27	3.27
$e \times 10^{-7}$	151	143	148	143	142	142

ing a total change in length of about 7 per cent. The micrometer caliper measurements before the test gave its length = 1.269 cm. while the day after the test it was 1.248 cm., a change of 1.65 per cent. Two days later the length was 1.253 cm. showing that it is gradually regaining its former length. This is thought to be due,

somewhat at least, to moisture, for when the air was first exhausted from the box there was a creeping of the fringes for about an hour in the direction indicating decreasing length. No. 2 was cooled but once and the curve is plotted from the single run, which is considered sufficiently accurate for the material. The total number of

FIBER. (COMPRESSED PAPER) NO. 1.

Cut Perpendicular to the Plane of the Sheet. Length = 1.248 *cm. Average Coefficient* = 472×10^{-7}. *Coefficients Corrected for Decreased Length at Lower Temperatures.*

Temp.	273°	233°	193°	153°	113°
	128.0	102.5	81.5	65.0	54.6
	125.0	102.5	82.1	65.5	54.5
Average	126.5	102.5	81.8	65.3	54.6
$e \times 10^{-7}$	691	562	449	358	300

FIBER. NO. 2.

Cut Perpendicular to Plane of Another Sheet. Length = 0.925 *cm. Average Coefficient* = 456×10^{-7}. *Coefficients Corrected at Lower Temperatures.*

	101.0	73.6	55.4	44.8	37.0
$e \times 10^{-7}$	745	543	408	331	273

FIBER. NO. 3.

Cut Parallel to Plane of Same Sheet from Which No. 2 was Cut. Length = 1.115 *cm. Average Coefficient* = 164×10^{-7}.

	41.7	31.8	24.2	20.1	16.7
$e \times 10^{-7}$	255	195	148	123	102

LEAD. CAST.

From Department of Chemistry, Supposedly Pure. Length = 1.5320 *cm. Average Width Fringes* = 125. *Density* = 11.51. *Average Coefficient* = $2{,}708 \times 10^{-8}$. *Coefficients Corrected for Change in Length.*

	64.80	63.88	60.98	58.31	55.92
	64.93	64.10	61.03	58.20	55.96
Average	64.87	63.99	61.01	58.26	55.94
$e \times 10^{-7}$	289	285	272	259	248

fringes down was 311.8 while there were only 305.3 up, showing a loss in length of but 2.1 per cent. as compared to 3.25 per cent. in the first cooling of no. 1. No. 3 was cut parallel to the plane of

QUARTZ. CRYSTALLINE.

Cut Parallel to Optic Axis. A Beautifully Clear Specimen Kindly Prepared and Loaned by Messrs. Tiffany and Co., New York. Length = 1.5264 *cm. Average Width Fringes* = 180. *Density* = 2.65. *Average Coefficient* = 5.686 × 10^{-8}.

	16.71	14.50	12.94	10.69	8.19
	16.99	15.14	13.26	10.58	8.18
Average	16.85	14.82	13.10	10.64	8.19
$e \times 10^{-8}$	7,542	6,642	5,865	4,762	3,664

HARD RUBBER.

Cut from Thick Sheet. Length = 1.680 *cm. Average Width Fringes* = 150. *Density* = 1.21. *Average Coefficient* = 546 × 10^{-7}. *Coefficients Corrected for Length Changes.*

	164.9	150.5	133.5	118.0	100.8
	166.5	150.3	134.4	118.5	100.6
Average	165.7	150.4	133.9	118.3	100.7
$e \times 10^{-7}$	673	613	546	485	413

SELENIUM.

Amorphous, from Eimer and Amend. Melted in a Porcelain Crucible and Cast in a Thin Sheet Zinc Mold. Zinc then Dissolved in Dilute Sulphuric Acid. No Apparent Formation of a Selenide. Length = 0.8561 *cm. Average Width Fringes* = 380. *Average Coefficient* = 3,732 × 10^{-8}.

	55.00	50.43	46.38	42.02	40.60
	55.01	50.67	46.24	42.01	40.50
Average	55.01	50.55	46.31	42.02	40.55
$e \times 10^{-7}$	439	404	369	335	324

ZINC. CAST.

From Eimer and Amend, Marked Chemically Pure. Length = 1.6985 *cm. Average Width Fringes* = 400. *Density* = 7.13. *Average Coefficient* = 264 × 10^{-7}. *Coefficients Corrected for Length Changes. Irregular in its Behavior. Two Specimens Tested. Both About the Same.*

Temp.	283°	263°	243°	223°	203°	183°	163°	143°	123°	103°
	36.4	36.9	36.9	36.6	34.8	33.4	31.5	31.4	26.3	21.3
	37.2	39.2	39.7	37.8	32.6	30.3	28.1	27.1	25.4	23.8
	37.1	37.8	38.6	38.4	35.5	32.8	31.5	30.0	29.2	22.9
	36.8	39.5	37.6	34.0	35.6	32.5	31.4	28.9	26.7	25.5
					32.9	33.0	28.3	27.8	25.8	24.5
Average	36.9	38.3	38.2	36.7	36.3	32.5	30.8	29.1	25.5	23.6
$e \times 10^{-7}$	297	308	308	297	291	260	246	233	204	190

the same sheet of fiber as no. 2 and a single decreasing temperature set of measurements was made. The average perpendicular expansion is about 2.83 times that of the parallel expansion.

The former curve for fused quartz is shown for comparison with the crystalline quartz.

Several of the specimens were tested in a magnetic field to see if any change in length was produced. The black glass mirror was supported upon a large electro-magnet and the specimen with its cover glass placed thereon under the tube *H*. The bismuth specimen especially was tested both in a field parallel to its length and perpendicular to its length ; but up to about one part in four million no change was observed in the field used, which was estimated to be at least 350 C.G.S. units by calculating backwards from the observations on iron to Bidwell's curves, as it was strong enough to show a decrease in length of more than a fringe in the iron specimen.

Selenium was tested to see if it changed length when illuminated. With the specimen in the box and at a constant temperature the cross wires were set over a dark band and the green light was cut off by interposing a screen above the lens *G*. After an interval of time the screen was removed and observation made to see if the fringe had changed position. No change was observable. Next a right angle prism was placed above the lens *G* so light from a gas burner could be thrown down into the tube to the top of the selenium projections upon which the top cover wedge rested. Again no change was detected. Now although the specimen had cooled slowly it is not necessarily assumed to be in its most sensitive condition to change its resistance when exposed to light, so a third trial was made. This time some selenium was placed upon a piece of glass and another piece of glass placed over it. This was placed upon a block of cast iron, covered over and slowly melted, kept in the melted state nearly an hour and then allowed to slowly cool. Some of the thin flakes were then placed upon black glass and covered with the wedge so that interference fringes were produced by the two glass surfaces. Both former tests of illuminating were then tried but with negative results. One hundredth of a half wave length could have been detected in either case but was not found.

The coefficients of expansion of platinum and crystalline quartz

offer good comparisons between this work and that of Karl Scheel,[1] who tested quartz parallel to its optic axis by this method and then by comparison tested other substances. Between + 16° and — 190° C. he gives figures from which the average coefficient of expansion of quartz for the temperature interval = 522×10^{-8}. Exterpolating my curve for the 10° from — 180° to — 190° and interpolating for the + 16° I find 60.98 fringes for the same interval, giving $e = 529 \times 10^{-8}$ or a result 1.3 per cent. higher than Scheel's. For platinum he gives 797×10^{-8}, while my result in the former paper for + 20° to 180° was given = 815×10^{-8} and exterpolating it as for quartz gives 805×10^{-8} or 1 per cent. higher. For the interval of + 16° to + 100° his value was 904×10^{-8}. Mine for the interval + 20° to + 100° is 906×10^{-8}. In a later paper, Scheel and Heuse[2] have redetermined platinum by means of a comparator and for the interval of + 16° to — 183° their value is 805×10^{-8}. Again exterpolating my curve for the 3° I get 808×10^{-8}, so that their last result is only 0.37 per cent. lower than mine.

Scheel,[3] has remeasured quartz glass by the direct ring method and finds its minimum length to be at — 84° C. instead of — 46° C., as in the case of the quartz cylinder previously examined. This is in close agreement with the statement in my previous paper, that the coefficient changed sign at about — 80° C.

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[1] Verh. d. Phys. Ges., 9, 1907.
[2] Phys. Zeit., 8, October, 1907.
[3] Verh. d. Phys. Ges., 9, 23, December, 1907.

SOME ELECTRICAL PROPERTIES OF SILICON.

By Frances G. Wick.

II. The Electrical Resistance of Silicon at Various Temperatures.

THE study of the resistance of the metallic silicon produced by the Carborundum Company is the second of a series of tests made upon a set of rods cast by H. E. Heath, the first measurements made being upon the thermo-electric behavior.[1] The experiments described in this paper include a study of the effect of temperature upon resistance through a range of 600 degrees.

The resistance of a number of the rods at room temperature was first measured by finding the fall of potential along a known length of the rod and comparing this with the potential drop around a known resistance.

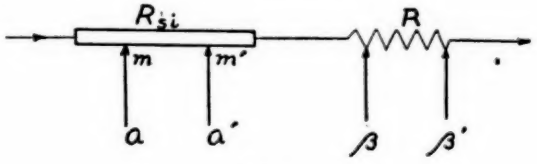

Fig. 1.

$$R_{Si} : R :: Pd_{aa'} : Pd_{bb'}.$$

The connections to the ends of the rods were iron wires fused to the silicon in an oxyhydrogen flame. The contacts mm' were made by means of brass springs. From the data obtained in this way and the dimensions of the rods the specific resistance of a number of rods was computed, *i. e.*, the resistance in ohms of a bar 1 cm. long and 1 sq. cm. in cross-section

$$R = K\frac{l}{q}.$$

[1] Wick, Frances G., Physical Review, Vol. XXV., No. 5, p. 382.

K = Specific resistance.

R = Resistance measured.

l = Length.

q = Area of cross-section.

TABLE I.

Length in cm.	Diameter in cm.	Resistance in Ohms.	Specific Resistance.	
4	.32519	1.6212	.033741	33741×10^{-6}
4.35	.319	2.984	.05498	54980 "
3.2	.3301	1.605	.04263	42630 "
20.3	.3167	20.41	.0793	79300 "
27.7	.3199	19.62	.05696	56960 "
27.2	.32045	26.17	.08061	80610 "

Table I. gives the results, which, it will be noticed, do not agree for any two of the rods. For all the specimens, however, the specific resistance is very high compared with that of other substances, that of copper being 1.3×10^{-6} and that of carbon from an Edison-Swan incandescent lamp being $4{,}000 \times 10^{-6}$. The average of the silicon rods measured was $58{,}036 \times 10^{-6}$.

TEMPERATURE AND RESISTANCE.

The effect of temperature upon the resistance of the rods was measured through a range of 600 degrees. The exceptionally large thermal E.M.F. between silicon and other metals, combined with the fact that the silicon rods were comparatively thick and very brittle, introduced considerable difficulty.

There was also trouble in making connections which were satisfactory at all temperatures. Brass springs were first tried with jaws fastening tightly around the silicon rod, then connections of fine copper wire closely wrapped around the rod. Neither of these methods was satisfactory, the contact being poor and variable especially at high temperatures. The rods were then copper plated in those places around which the wire was wrapped but at high temperatures the copper plate loosened. Finally a contact which was satisfactory at all temperatures was made by fusing iron wire into the silicon rod in an oxyhydrogen flame. With this sort of con-

nection the results obtained at ordinary temperatures were the same as with the wrapped or spring connections, showing that the resistance of the silicon was not altered during the process of fusion.

The method of measuring resistance with change in temperature was as follows:

Short iron wires were fused to the rod at the ends A and B (Fig. 2) and at the points C and D. Copper constantan thermo-junctions were bound tightly to the rod at E and F, the other junctions of these pairs (x and y) being kept at 0°. The ends of the rod were then covered with plaster of Paris to keep the wires from breaking off. The copper wires used for connections which were subjected to high temperatures were insulated in pipe stems.

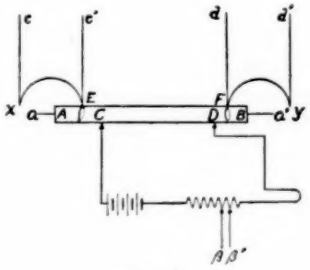

Fig. 2.

For the measurement of resistance at high temperatures the rod was placed at the center of a long heating coil made by winding iron wire evenly upon a porcelain tube, the inner diameter of which was about 1.5 cm. By varying the current through the coil, temperatures between 0° and 400° were obtained. The position of the rod in the tube was adjusted until the thermo-junctions indicated the same temperature at both ends.

By connections with potentiometer leads the fall of potential was measured between terminals aa', bb', cc' and dd', the connections being changed quickly by means of mercury cups. aa' gave the fall of potential along the rod, bb' that around a known resistance cc' and dd' gave the E.M.F. of the thermo-junctions from which the temperature was determined by reference to a calibration curve previously taken.

For temperatures below zero the rod with its connections was immersed in CO_2 and ether, and finally in liquid air.

To eliminate thermal effects, a system of reversing keys was arranged so that the current through the silicon rods and the potentiometer were reversed at the same time. Direct and reversed readings were taken in all cases and the average computed. Thermal effects were eliminated to such an extent that the two

TABLE II.

R	Potentiometer Readings for Copper Constantan Junctions.			Temperature from Calibration Curve.	Fall of Potential. Aronnd Silicon.			Fall of Potential. Around R.			R_{Si}	K
	(1)	(2)	Av.		Direct.	Reversed.	Av.	Direct.	Reversed.	Av.		
5	87	88	87.5	23	1647.1	1643	1645.05	1742.3	1742.3	1742.3	4.72	.0437
	323	324.5	323.75	78.5	1515.4	1511	1513.2	1738.2	1739	1738.6	4.35	.04029
	552	553	552.5	126	1439	1460	1449.5	1739	1739	1739	4.16	.038536
	774	778	776	172	1492	1348	1420	1743	1745	1744	4.07	.03798
	507	512	509.5	118	1470.9	1439.5	1455.2	1735.2	1735.2	1735.2	4.2015	.03892
	1032	1037	1034.5	216	1345.5	1473.6	1409.55	1742	1742	1742	4.103	.038
	79.2	80.8	80	21	1647	1642	1645.5	1734	1735	1734.5	4.66	.043168
	1442	1440	1441	292	1290	1568	1429	1733	1736	1734.85	4.12	.03816
	1528.5	1531.1	1529.55	306	1380	1477	1428.5	1737	1739	1738	4.11	.03809
	1765.8	1769	1707.8	340	1460	1500	1455	1747	1747	1747	4.164	.038573
	2036	2034	2035	380	1447	1496.6	1471.8	1729.2	1731	1730.1	4.285	.03971

Diameter of rod = 3.24 mm.
Length around which fall of potential 25 as measured = 8.9 cm.

readings differed very slightly, as may be seen in Table II. which is a sample table giving all readings as they were actually taken.

Fig. 3 gives curves from data obtained in a similar way showing the effect of temperature upon resistance. The dimensions of the

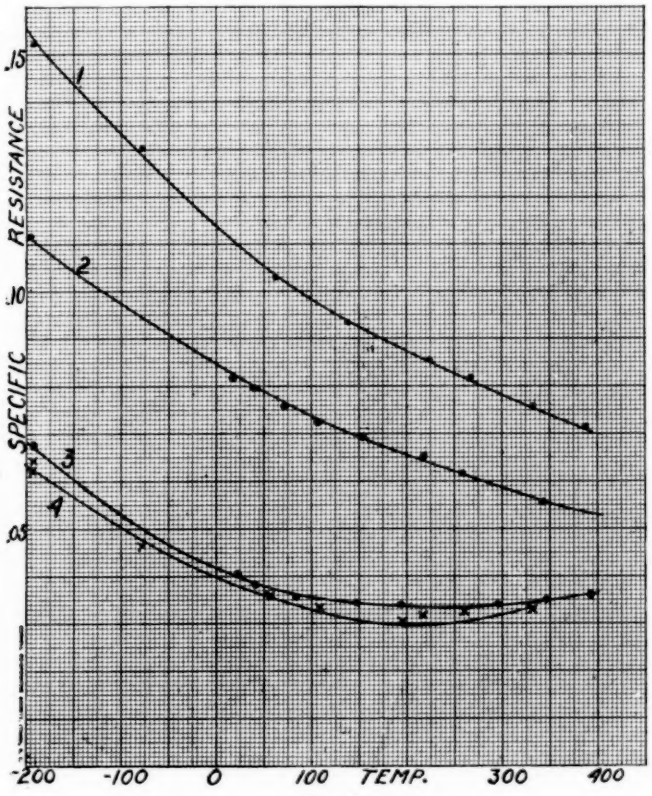

Fig. 3.

rods are given in Table III. Fig. 4 gives the change in specific resistance per degree for rods 1, 2, 3 and 4 computed from the corresponding curves given in Fig. 3 by taking the slope at different points.

These results indicate a wide variation not only in the absolute values of specific resistances but also in the shape of the curve representing the change in resistance with temperature.

The rods seem to belong to two different groups. For one the

temperature coefficient of resistance is negative throughout the range measured. Rods 1 and 2 belong to this group.

TABLE III.

Rod.	Diameter.	Length along which Fall of Potential Was Measured.
1	3.09 mm.	8.3 cm.
2	3.34	10.5
3	3.24	7.8
4	3.26	5.5

For the other the coefficient is negative from the lowest temperature measured to about 200°. At this point a change takes place and the coefficient becomes positive.

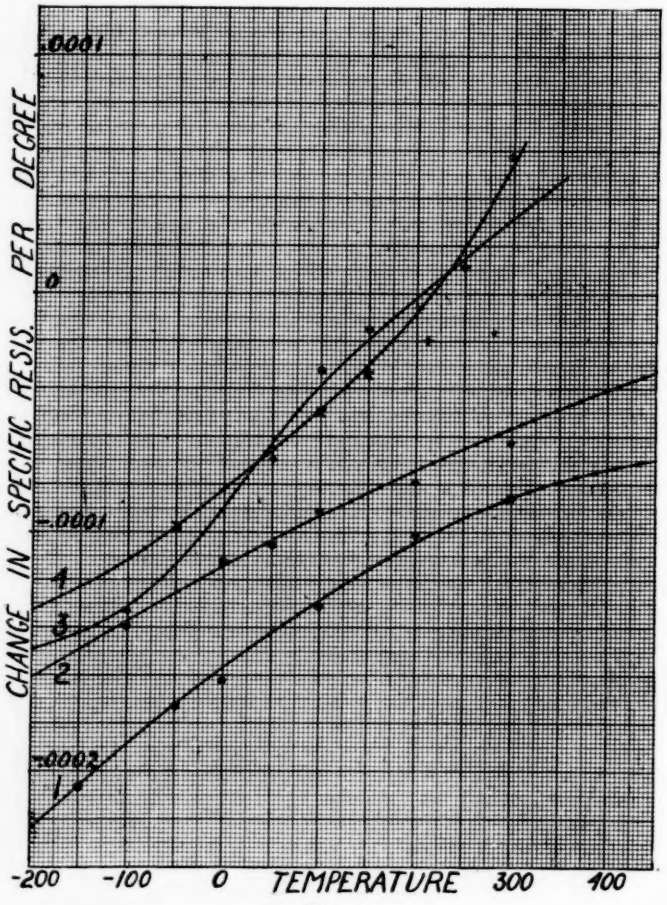

Fig. 4.

Rods 3 and 4 are two of the rods which in previous tests showed comparative agreement in thermal E.M.F. Rod *A*, was later measured for thermal E.M.F. and the value obtained was about half that for the other rods.

These results show a lack of uniformity in the resistance of different specimens of the same lot as well as in the effect of change in temperature upon this resistance indicating that the resistance of metallic silicon is very sensitive to changes in purity, physical constitution, crystalline structure, or variations of some other sort the exact nature of which, in this case, is not known.

The substitution of silicon for other resistance where electrical heating is required has been considered, since the resistance, specific heat and emissive powers are all high. But the extreme brittleness of the substance offers so much difficulty that it has not yet been successfully used.

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HEAT DEVELOPED IN A MASS OF THORIUM OXIDE, DUE TO ITS RADIOACTIVITY.

BY GEORGE B. PEGRAM AND HAROLD W. WEBB.

IN a former short notice[1] the authors described an experiment for the purpose of detecting and measuring the rate at which heat is developed in a mass of thorium oxide, on account of its radioactivity. The present paper gives the results of more careful experiments for the same purpose.

The method of the experiment was briefly as follows: The thorium oxide (not pure, but old Welsbach mantles ground up, for which we are indebted to Mr. H. Lieber) was contained in a five liter spherical Dewar bulb, suspended in a metal cylinder, which in turn was suspended in an ice-bath. By means of thermo-elements with one set of junctions in the thorium oxide, the other set against the top of the metal cylinder, the temperature difference between the thorium oxide and the ice-bath was measured. Heat was then produced in the bulb at a known rate by sending a current through wires laid in the thorium oxide, and the temperature difference determined after a constant value had been reached. As the temperature differences were very small they were assumed directly proportional to the rate of heat development, and so from the temperature differences and the known rate of heat development by the current the rate of heat development by the thorium oxide was estimated.

In the Dewar bulb, *D*, Fig. 1, were 4.1 kg. of the thorium oxide, which filled it a little way up the neck. The metal cylinder, *V*, within which the bulb hung on cords from the top, with a clearance of 1 cm. between it and the cylinder wall, was a piece of steel pipe 53 cm. long, 27 cm. diameter, with brass caps at top and bottom. The purpose of the cylinder was in the first place mechanical protection to the Dewar bulb, but to insure that all was dry within the bulb,

[1] Science, May 27, 1904.

as well as to decrease the rate of heat loss from the bulb, the cylinder was made air-tight and exhausted to a pressure of .4 mm. Hg. Difficulty was experienced at first in getting the cylinder air-tight, but by having the steel pipe heavily galvanized inside and out, and using rolled brass caps, the difficulty was overcome. The caps of the cylinder were provided with grooves to receive the rim of the pipe and the joints were made tight with a cement of gutta-percha and rosin. Then it was so tight that no change in pressure as indicated on an ordinary mercury vacuum gauge could be detected in nine months.

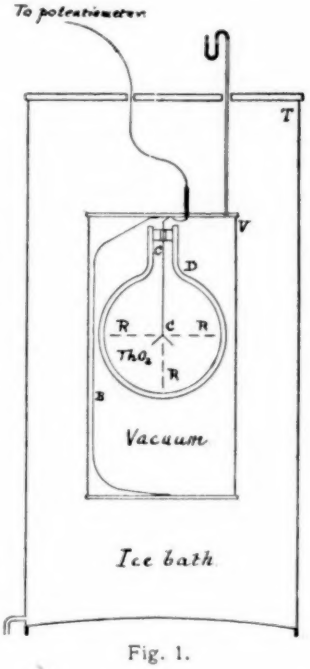


Fig. 1.

The ice-bath was a cylindrical sheet-iron tank, *T*, 100 cm. deep, 50 cm. in diameter (giving a clearance of 11 cm. between the cylinder and the wall of the tank), with a small outlet at the bottom so that water from the melting ice did not accumulate in the ice-bath. Underneath and around the tank saw-dust was packed, nowhere less than 12 cm. deep. The tank had a wooden cover, and pieces of thick felt were laid over it. Hygeia (artificial) ice was used, broken into pieces of 2 to 20 c.c. size. About 15 kg. were used daily. As appears in the table given later, thermo-element readings generally showed a small temperature difference between the top and the bottom of the cylinder, the bottom being warmer.

The thermo-elements were of iron wire 0.17 mm. diameter, and constantan wire 0.11 mm. diameter, with junctions soldered. The wire was silk insulated and the bared junctions covered thinly with paraffin. There were three sets of thermo-elements. The principal set, of five couples, *C*, which we shall refer to as the *central couples*, had one set of junctions arranged radially at the center of the mass of thorium oxide, so that they were all within 2 cm.

of the center of the mass. The other junctions were outside the bulb, clamped tightly, with only a thin sheet of mica between for insulation, against the top of the metal cylinder. For a check on the working of these elements, a lead-wire was put in so that the electromotive force of the five couples could be measured in two parts, one of two, the other of three couples. As the sum of these readings agreed well with the reading for all five together, no further use was made of the readings by parts.

The second set, of six couples, R, which we shall refer to as the *radial couples*, was also arranged with one set of junctions within 2 cm. of the center of the thorium oxide, and the other junctions were within about 2 cm. of, the wall of the containing bulb, four radiating symmetrically in the horizontal plane, and the remaining two in the vertical axis. These gave by their readings approximately the temperature difference between the center and the outside of the mass of thorium oxide. Furthermore, this set of wires also served as the conductor through which a current was passed to introduce heat into the bulb at a known rate. As the current was quite small, the heating or cooling at the junctions, due to the Peltier effect, was considerable in proportion to the Joule heat developed, and to obviate the effect of this the direction of the current was reversed twice a minute by clockwork.

The third set of five thermo-elements, B, served as a check on the uniformity of temperature of the metal cylinder. One set of junctions was at the top of the cylinder, the other set lay on the bottom of it.

Copper lead-wires to the thermo-elements ran out through a tube inserted in the top of the metal cylinder, in which tube the wires were sealed with wax and then ran on well out of the bath (through a piece of rubber pressure tubing). The wires were also sealed with wax in the outer end of this rubber tube, to prevent moisture condensing on them in the cold end of the tube. The lead-wires led to a small switch-board of copper blocks with mercury cups, so arranged that all desired connections to potentiometer, etc., could conveniently be made with copper links between the cups. The switch-board was enclosed in a wooden box, to prevent sudden temperature variations.

The electromotive force of the thermo-elements was measured by

balancing against a straight wire potentiometer. The potentiometer wire was a meter length of copper wire, resistance .087 ohm, connected in series with a resistance of 30,000 ohms and a cell of 1.46 volts E.M.F. The fall of potential for 1 cm. of the potentiometer wire was then $.042 \times 10^{-6}$ volts. All conducting parts of the potentiometer were of copper. Only in the mercury cups and the galvanometer was any other metal in the circuit outside of the ice-bath. The galvanometer used was a special high sensibility suspended coil galvanometer, made by the Weston Electrical Instrument Company. Its resistance was 31.2 ohms, and 1 scale division corresponded to 8.2×10^{-9} volts across its terminals. The galvanometer was also enclosed in a wooden box to equalize its temperature, as its binding-posts were aluminum.

The mass of thorium oxide was not of the same temperature throughout, being warmer towards the center of course. Let us suppose that the distribution of temperature, in the case where the heating current was flowing, was the same (except for a constant factor) as in the case of no current, and that we knew the temperature above the ice-bath of any point of the mass, say a point near the surface, in each case. Then we could say that the ratio of the rates of heat development in the two cases was the same as the ratio of the temperatures. The radial couples give us the temperature difference between the center of the mass and the region within 2 cm. of the outside. Subtracting the reading of the radial couples from that of the central couples gives the temperature difference between that region 2 cm. deep in the mass and the ice-bath. Now with no heating current the temperature difference between the center and this outer region is about one fourth of the difference between this region and the ice-bath, while with current it is about one third. Hence it is evident that the heat was more concentrated toward the center of the mass in the second case, which is a necessary consequence of the "star" arrangement of the heating wires; but it is also evident that this difference in distribution of temperature makes no great error in the result, if we say that the rate of heat development in the mass in the two cases is proportional to the temperature elevation of the thorium oxide at a depth of 2 cm. Certainly the error from this is less than the difference between one fourth and onethird, for this part of the mass for which the temper-

atures are compared, namely, the part at a depth of 2 cm. from the outside, is much nearer the average temperature than the center.

As the cooling and heating due to the Peltier effect at the respective junctions of the radial set of thermo-elements, the wires of which were used as the heating circuit, showed itself very plainly in the readings of the electromotive force from these junctions when readings were taken immediately, it was necessary to wait some time after the current was cut off before the readings were free from the disturbance due to this very local heating and cooling. Forty-five minutes was, however, quite sufficient. In the meanwhile the bulb as a whole had cooled off a little. The rate of cooling was ascertained by a series of measurements extending over a considerable period after the current was cut off, and it was found that the cooling was about at the rate of half in 35 hours. We have, therefore, made corrections to the observed readings, taking account of the cooling in the time between the cessation of the heating current and the time when each reading was made.

No correction has been made for the small difference of temperature between the top and bottom of the cylinder, as this was found to have no appreciable effect on the results. The readings of the cylinder couples were used simply as a check on the constancy of the temperature of the ice-bath. No measurements of the temperature of the thorium oxide were taken, when the cylinder couples showed large differences of temperature or rapid variation.

The effect of the concentration of heat toward the center when the heating current was on can be seen easily in table (A). There the readings of both the central and radial couples drop rapidly at first due to the diffusion of heat from the central part, yet the difference between the readings remains about constant. It is not this rate of drop that is used for making the correction just mentioned, but the much longer rate of cooling of the bulb as a whole.

Of course to get rid of disturbing electromotive forces in the circuit outside of the ice-bath, the mean of the potentiometer readings with connections first one way, then reversed, was always taken. There was sometimes no difference between the two readings, sometimes again it was as much as 60 per cent. of the smaller reading. Only the means are given in the table.

The actual temperature difference measured between the thorium oxide and the ice bath, as calculated from the thermo-electric constant was, with no heating current, 3.4×10^{-30} C.

Table of Results.

(*A*) *With Heating Current*, 15.25×10^{-5} *Watts Inside Bulb.*

Time.	Wait Between Cessation of Current and Reading.	Numbers are cms. on Potentiometer Wire. 1 cm. = $.042 \times 10^{-6}$ Volt.				
		Central Couples.	Radial Couples $\times \frac{8}{6}$.	Difference.	Corrected for Wait.	Cylinder Couples.
May 29, 8.45 P. M.	90 min.	134.2	54.0	80.2	82.7	
9.30 P. M.	135 "	127.5	47.3	80.3	83.6	12.0
10.30 P. M.	195 "	120.0	43.7	76.3	81.5	8.8
11.50 P. M.	275 "	113.8	37.5	76.3	83.6	11.1
May 30, 9.45 P. M.	60 "	136.8	52.7	84.1	85.8	7.5
10.45 P. M.	120 "	126.6	43.0	83.6	87.1	6.6
May 31, 7.00 P. M.	75 "	141.2	53.5	87.7	90.0	3.6
8.00 P. M.	135 "	134.8	47.0	87.8	92.0	4.5
9 00 P. M.	195 "	124.8	41.5	83.3	89.0	3.7
June 1, 7.30 P. M.	80 "	133.5	52.3	81.2	83.4	8.1
8.20 P. M.	130 "	126.5	47.5	79.0	82.6	7.2
9.00 P. M.	170 "	122.7	43.0	79.7	84.4	8.7
June 2, 10.55 P. M.	150 "	125.2	46.3	78.9	83.0	7.5
June 3, 8.15 P. M.	60 "	142.2	57.5	84.7	86.4	7.8
9.20 P. M.	125 "	132.7	46.5	86.2	90.0	—
10.10 P. M.	175 "	126.0	43.5	82.5	87.6	7.2
June 4, 1.40 P. M.	40 "	145.8	62.5	83.3	84.4	—
2.30 P. M.	90 "	131.4	52.3	79.1	81.4	—
3.30 P. M.	150 "	123.4	45.0	78.9	83.0	6.6
4.30 P. M.	210 "	119.4	40.5	78.9	84.8	—
					Mean 85.4	

(*B*) *Thorium Oxide with No Heating Current.*

June 10, 10.00 A. M.		30.0	8.2	21.8		5.7
" 11, 7.00 A. M.		31.2	8.5	22.7		6.0
" 13, 9.00 A. M.		27.6	8.0	19.6		5.1
" 13, 10.00 P. M.		26.1	8.2	17.9		1.5
" 14, 1.30 P. M.		28.9	8.0	20.9		3.9
" 15, 1.15 P. M.		26.4	9.7	16.7		4.8
" 16, 10.30 A. M.		30.6	7.0	13.6		2.4
" 16, 5.00 P. M.		30.0	7.9	22.1		4.8
" 17, 1.45 P. M.		26.7	6.9	19.8		6.6
				Mean 19.5		

TABLE OF RESULTS. — *Continued.*

(*C*) *With Heating Current*, 7.03×10^{-5} *Watts Inside Bulb.*

Time.	Wait Between Cessation of Current and Reading.	Numbers are cms. on Potentiometer Wire. 1 cm. = $.042 \times 10^{-6}$ Volt.				
		Central Couples.	Radial Couples $\times \frac{8}{5}$.	Difference.	Corrected for Wait.	Cylinder Couples.
June 19, 10.30 P. M.	180 min.	66.9	22.8	44.1	46.8	3.9
" 21, 10.30 P. M.	60 "	79.8	31.2	48.6	49.6	(2.1)
" 21, 11.00 P. M.	240 "	69.3	21.5	47.8	51.9	—
					Mean 48.8	

(*D*) *Thorium Oxide with No Heating Current.*

June 28, 9.00 A. M.		26.4	4.8	21.2		3.0
1.00 P. M.		28.2	5.5	22.7		—
8.00 P. M.		29.4	5.8	23.6		0.9
" 29, 9.00 A. M.		26.5	5.5	21.0		0.9
2.30 P. M.		27.0	5.5	21.5		1.0
July 1, 9.00 A. M.		23.1	6.8	16.3		5.0
				Mean 21.0		

(*E*) *With Heating Current*, 7.03×10^{-5} *Watts Inside Bulb.*

July 2, 1.45 P. M.	45 min.	74.1	24.2	49.9	50.7	
" 3, 2.30 P. M.	90 "	71.1	29.2	47.9	49.3	
" 4, 2.00 P. M.	90 "	71.1	23.5	47.6	49.0	
					Mean 49.7	

The resistance of the radial set of couples was 32.8 ohms (including lead-wires to neck of bulb), of the central set 87.0 ohms (without lead-wires).

Setting together the results from the readings given in the tables, we have for the means, in terms of cms. on the potentiometer wire (1 cm. corresponding to $.042 \times 10^{-6}$ volt)

Without heating current,		19.6
With heating current,	7.03×10^{-5} watts	49.2
With heating current,	15.25×10^{-5} watts	85.4

That is,

(*a*) Due to radioactivity of the ThO_2,		19.6
(*b*) Due to 7.03×10^{-5} watts,		$49.2 - 19.6 = 29.6$
(*c*) Due to 15.25×10^{-5} watts,		$85.4 - 19.6 = 65.8$

By proportion between (*a*) and (*b*) the rate of heat liberation by the thorium oxide is 4.66×10^{-5} watts; from (*a*) and (*c*) it is

4.54×10^{-5} watts, with a mean for the two of 4.55×10^{-5} watts. The figures for the effect of 15.25×10^{-5} watts and 7.03×10^{-5} watts correspond very well, as the ratio of the watts is 2.17, and the ratio of the temperature differences $68.8/29.6 = 2.22$. Per gram of the thorium oxide used the rate of heat liberation is $4.55 \times 10^{-5}/4,100 = 1.11 \times 10^{-8}$ watts, a little more than a tenth of an erg per second per gram. Expressed in units that have been often used by Rutherford, it is 9.60×10^{-6} gram calories per hour per gram. Radium bromide in its equilibrium state gives 65 calories per hour per gram. Hence, as measured by its total energy development, radium bromide in its equilibrium state is 6.8×10^{6}, nearly seven million times as active as this thorium oxide.

This thorium oxide was not in its equilibrium state, but much below it in activity. Dr. Otto Hahn, of the University of Berlin, has been kind enough to compare by ionization test the activity of our oxide with that of specimens of which he knows the history, time elapsed since preparation especially, and it comes out that our oxide, though a chemical analysis shows it to be 90 per cent. ThO_2, has an activity only 46 per cent. of ThO_2 in its equilibrium or mineral state, which is what was to be expected if, as probable, the thorium was separated from the mineral eight or ten years ago, and contains five to ten per cent. of impurities. It is of interest to estimate from our results with this material what the rate of heat development in a mass of thorium oxide is when all the disintegration products are present in equilibrium amounts. On the well supported view that the heat development comes almost wholly from the kinetic energy of the α particles, we cannot say in general that the heating and ionizing effects are proportional when comparing mixtures of active substances sending out α particles of different ionizing ranges, for the ionizing power of an α particle is not proportional to its kinetic energy. We may discuss the present case as follows.

In the the thorium oxide as used was, besides thorium itself, meso-thorium and radio-thorium in about half the amounts for equilibrium with the thorium, and all the products subsequent to RaTh in equilibrium with it, since RaTh has a very slow decay rate compared with any of the subsequent products. The thorium itself gives α rays of short range (probably about 3 cm. in air),

the meso-thorium gives β rays only, which may be neglected in comparison with the α rays, while the RaTh and subsequent products give five sets of α rays, some of long range. If now we were to compare the total ionizing action of two samples of material, both containing RaTh and subsequent products in equilibrium, but in different amounts, the comparison would give us simply the relative amounts of the same radioactive substance in the two samples, and therefore the ratio of the rates of heat development in the two substances would be the same as the ratio of the ionizing actions. What we have actually done is to compare the ionization due to the thorium in our substance plus half the equilibrium amount of subsequent products, with the ionization due to thorium plus the full equilibrium amounts of all its disintegration products. Since, however, the α rays of the thorium itself are of short range, and therefore furnish in either case only a small part of the ionization or of the heat, compared to the five sets of more penetrating α rays in the RaTh and subsequent products, it is evident that the rates of heat development will be very nearly in the ratio of the ionizing effects, even if the ionization due to α particles of different range is not closely proportional to their energy. [The way will be open to a more exact discussion of the matter when the range of the α particle from thorium is determined.] We may say, therefore, that the rate of energy liberation in the material we used is 46 per cent. of the rate of energy liberation in thorium oxide with all its disintegration products present in equilibrium amount. Finally, that the rate of heat development in old or mineral thorium oxide is $.96 \times 10^{-5}/.46 = 2.1 \times 10^{-5}$ calories per hour per gram. Radium bromide in its equilibrium state (as far as radium C) liberates heat at 3.2 million times this rate.

PHŒNIX PHYSICAL LABORATORY,
COLUMBIA UNIVERSITY,
March 7, 1908.

A STUDY OF DISPERSION IN HIGHLY ABSORBING MEDIA BY MEANS OF CHANNELED SPECTRA.[1]

By. S. R. Williams.

THE discovery of the phenomenon of anomalous dispersion is, by some authors, credited to Fox Talbot[2] about the year 1840, but not until 1862 did le Roux[3] give quantitative values of the indices of refraction of iodine vapor and show that the red rays were more refrangible than the blue.

This remarkable phenomenon seems to have aroused little interest at the time and was for a while forgotten. Later, in 1870, Christiansen[4] published the results of his investigations on the dispersion of saturated solutions of alcohol and fuchsin, which substantiated the work of le Roux and showed conclusively that certain substances possess the property of anomalous dispersion in the visible portion of the spectrum. This subject has since proved a fruitful field of investigation. Kundt[5] who studied the dispersion of strongly absorbing media found that all bodies showing surface color possessed what we ordinarily call anomalous dispersion.

He also observed that in solutions the anomalous dispersion decreases continuously with decrease in concentration. In this work Kundt applied to the investigation of anomalous dispersion the very ingenious method of crossed prisms, due originally to Newton. This method has been improved by Wood[6] who replaced one of the prisms by a small grating.

Among the later and more comprehensive studies of anomalous dispersion and absorption in solutions may be mentioned the inves-

[1] Presented before the American Physical Society, February, 1908.

[2] Proc. Roy. Soc., Edinburgh, 1870–71.

[3] le Roux, Comp. Rend., 55, p. 126, 1862.

[4] Christiansen, Pogg. Annal., 141, p. 470, 1870; 143, p. 250, 1871; 146, p. 154, 1872.

[5] Kundt, Pogg. Annal., 142, p. 163, 1871; 143, p. 259, 1871; 144, p. 128, 1872; 145, pp. 67, 164, 1872.

[6] Wood, Phil. Mag., June, 1901.

tigations of Stöckl,[1] Katz[2] and Claes;[3] while for solid substances Pflüger,[4] Wood[5] and Cartmel[6] have made important contributions.

After looking over the various researches just mentioned it appeared that anomalous dispersion could be studied advantageously by means of channeled spectra, the use of which in determining refractive indices was given in a former paper.[7] This method admits of a detailed study of closely adjacent portions of the spectrum as for instance the D_1 and D_2 lines, whereas an investigation of dispersion in highly absorbing media by means of prisms necessitates very small refracting angles, and consequently small dispersion.

The essential features of the method of determining refractive indices by means of channeled spectra were as follows: Light fell upon a thin wedge-shaped film of the substance to be investigated and the two beams, reflected from the front and from the rear surface, illuminated the slit of a grating spectrometer. In the resulting spectrum destructive interference occurred for those wave-lengths for which the difference in path was an odd number of half wave-lengths, according to the formula for normal incidence,

$$\frac{(2N+1)}{2} = \frac{2\mu d}{\lambda},$$

where N is the difference in path in wave-lengths, μ the index of refraction, d the absolute thickness, and λ the wave-length.

Hence if the interference bands from two solutions are compared in the same part of the spectrum, we can study their relative dispersion by observing the relative positions of bands in the two spectra. The comparison is most easily made by introducing the solutions into the same cell, one solution above and the other below a dividing strip which runs lengthwise of the cell. In the photographs the arrow at the left of each picture indicates the position of the dividing strip. In the spectrum the upper half of the interference bands will be due to one substance and the lower half due to the other.

[1] Stöckl, Inaug. Diss., München, 1900.
[2] Katz, Inaug. Diss., Erlangen, 1898.
[3] Claes, Wied. Annal., 3, pp. 384-414, 1878.
[4] Pflüger, Wied. Annal., 65, p. 203, 1898.
[5] Wood, Phil. Mag., June, 1901; Phil. Mag., 1903.
[6] Cartmel, Phil. Mag., 1903.
[7] PHYS. REV., p. 280, April, 1904.

Some of the solutions studied in this paper by this method were the alcoholic solutions of fuchsin, cyanine, litmus, eosin and naphthalin red, which Mr. F. J. Bates[1] used in studying rotatory dispersion.

The formulæ for rotatory dispersion, as derived from various dispersion theories, indicate that, if in any medium an anomalous rotation of the plane of polarization occurs, anomalous dispersion must also be present. Mr. Bates made his solutions of the same concentration that a number of other investigators had previously used, and although they found anomalous rotatory dispersion, he was unable to find it when corrections were made for errors in illumination which he has discussed in his paper. To substantiate his results further, Mr. Bates asked that the solutions which he was using be tested for anomalous dispersion by means of channeled spectra, as this was a very sensitive method for detecting small changes in dispersion. Accordingly the interference bands from thin films of these solutions were compared with those from a thin film of absolute alcohol,[2] since in the study of the rotatory power these solutions were compared with that solvent. None of the solutions showed any shift in the bands with respect to those from alcohol, and only one photograph is shown, No. 1, Plate I., for a solution in which the concentration was 0.0000244 gram of fuchsin to one gram of alcohol.

The values of the indices of refraction of alcohol as measured by means of channeled spectra were given in a former paper[3] and show no anomalous dispersion. Consequently the absence of a shift between the bands from alcohol and those from the solutions indicates the absence of anomalous dispersion in the solutions, which corroborates the work of Mr. Bates.

Not finding any shift in one set of bands with respect to the other, in the solutions at these concentrations, it was of interest to determine the concentrations for which shifts occurred, and the nature of these shifts. This was done for fuchsin by making up solutions of two, three, and four times the concentration of the

[1] F. J. Bates, Wied. Annal., 317, p. 1091, 1903.

[2] For a discussion of the detection of anomalies when two dispersive substances are compared, the reader is referred to a paper by the late Professor D. B. Brace, PHYS. REV., November, 1905.

[3] Loc. cit.

original solutions. Photographs are shown in Figs. 2, 3 and 4 of Plates I. and II.

In Fig. 2 there is an irregularity in some of the bands between D_1 and β_1 but not enough to prove an actual shift. In Figs. 3 and 4 the shift is unquestionable, Fig. 4 showing a greater displacement than Fig. 3. Between wave-lengths 5528.6 and 5896.1, the lower half of the bands due to the fuchsin solutions appear to be displaced toward the red end of the spectrum with respect to the upper half of the bands from the alcohol films. Between wave-lengths 5447.0 and 4383.7 the shift is in the opposite direction.

The orders of interference were determined for various wave-lengths in the two media, the difference for any particular wave-length giving the shift in fractions of a band.

In the following tables the wave-lengths are given in the first column, the orders of interference for alcohol and fuchsin solutions in the second and third respectively and their differences in the fourth.

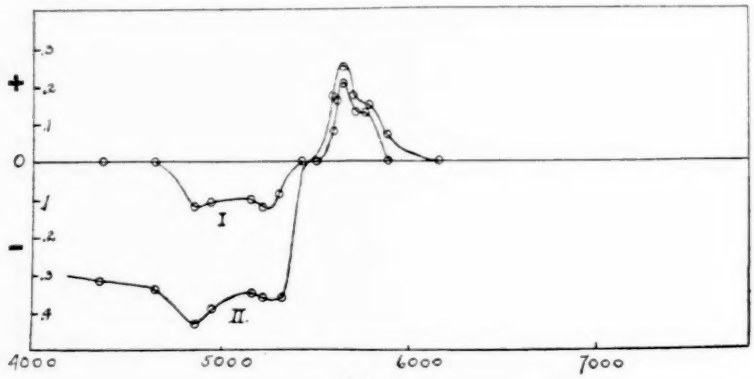


Fig. 1.

In curves I. and II., Fig. 1, are plotted with wave-length, the shifts for the two concentrations corresponding to photographs 3 and 4 respectively (Plate II.). The shift has been drawn on a large scale as compared with the wave-length and of course the errors in measuring the shift are correspondingly magnified.

An inspection of the curves reveals some interesting results. First, the dispersion curves of fuchsin solutions have maxima and minima which other investigators, using prismatic methods, have overlooked. Secondly, if we had an equally sensitive method for measuring the absorption of fuchsin in this region, we should find

the absorption band made up of several maxima very close together, while in the absorption curve of fuchsin given by Cartmel[1] it appears to be a broad single maximum.

TABLE I.

Fuchsin Dissolved in Alcohol. 0.000072 *Part of Fuchsin to One of Alcohol by Weight. Compare with Photograph* 3, *Plate II.*

Wave-length.	Alcohol.	Fuchsin.	Shift.
6.1975 × 10^{-5} cms.	2551.71	2551.71	.00
5.8961 "	2657.50	2657.50	.00
5.7912 "	2706.50	2706.63	.13
5.7250 "	2744.64	2744.77	.13
5.6589 "	2770.87	2771.07	.20
5.6169 "	2792.74	2792.90	.16
5.6031 "	2798.99	2799.07	.08
5.5286 "	2837.35	2837.35	.00
5.4470 "	2880.50	2880.50	.00
5.3285 "	2945.93	2945.84	−.09
5.2270 "	3004.33	3004.21	−.12
5.1728 "	3036.42	3036.32	−.10
4.9577 "	3170.88	3170.77	−.11
4.8615 "	3235.00	3234.88	−.12
4.6548 "	3372.43	3372.43	.00
4.3837 "	3578.83	3578.83	.00

TABLE II.

Fuchsin Dissolved in Alcohol. 0.000096 *Part of Fuchsin to One of Alcohol by Weight. Compare with Photograph* 4, *Plate II.*

Wave-length.	Alcohol.	Fuchsin.	Shift.
6.5630 × 10^{-5} cms.	2362.89	2362.89	.00
6.1035 "	2544.00	2544.00	.00
5.8961 "	2635.27	2635.34	.07
5.7912 "	2684.00	2684.15	.15
5.7095 "	2721.91	2722.08	.17
5.6589 "	2747.91	2748.16	.25
5.6169 "	2769.08	2769.25	.17
5.6031 "	2776.00	2776.17	.17
5.5286 "	2813.18	2813.18	.00
5.4470 "	2856.81	2856.81	.00
5.3285 "	2921.38	2921.02	−.36
5.2270 "	2979.00	2978.64	−.36
5.1728 "	3010.94	3010.59	−.35
4.9577 "	3144.42	3144.03	−.39
4.8615 "	3207.88	3208.45	−.43
4.6548 "	3344.27	3343.93	−.34
4.3837 "	3548.73	3548.41	−.32

[1] Loc. cit.

Moreover, the point where the dispersion curve of the solution of fuchsin cuts the dispersion curve of alcohol, does not agree with that found by Stöckl. He showed that all concentrations of fuchsin solutions have the same point of intersection of the dispersion curve as solid fuchsin, although he also showed that with increase of concentration, there is a shifting of the maximum of absorption toward shorter wave-lengths. Of course any discrepancy between observations may be attributed to differences in the samples of fuchsin as it is a compound which varies considerably. The fuchsin used in this work was the same as that employed by Bates and Cartmel, and was obtained from the same source as that used by Pflüger, to whose work Stöckl referred his results.

In the photographs the smallest shift of bands measured was .07 of a band, a shift apparent to the naked eye. For the order of interference used this represents a change of about 0.00008 in the index of refraction of the solution as compared with that of alcohol, the indices of refraction of which were given in a former paper. Even in the solution whose concentration is four times that of the original, no anomalous dispersion is present.

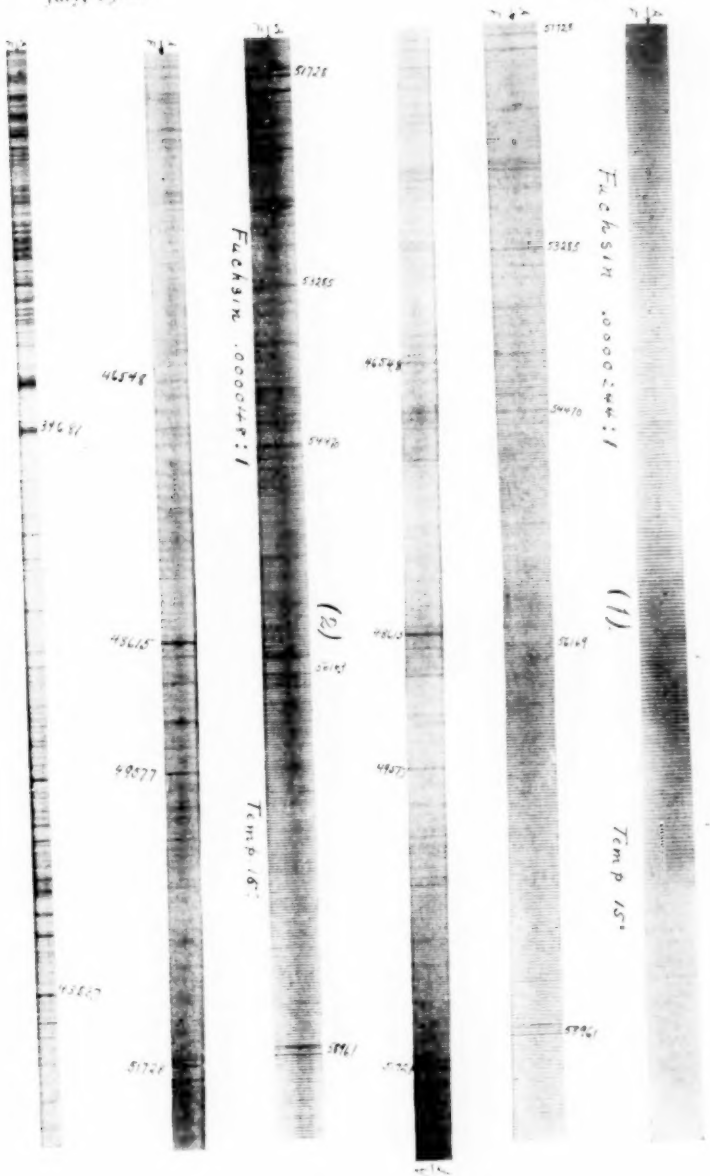
As a method of comparison, channeled spectra offer decided advantages in studying the relative dispersion in two substances. The solutions are introduced into the cell and any slight shift of one set of bands with respect to the other is at once noticeable as one passes through the spectrum with the observing telescope. If quantitative measurements are wanted, it is found most convenient to photograph the spectra and afterwards measure the photographs.

It is hoped that this method may further be used in studying the influence of changes, both in the index of refraction and of the concentration of the solvent, on the shift of the absorption bands, a subject concerning which very little is known either theoretically or experimentally.

The experimental data for this paper were obtained in the Physical Laboratory of the University of Nebraska, at the suggestion of the late Professor Brace, of whose interest and advice in this work the author has very pleasant recollections.

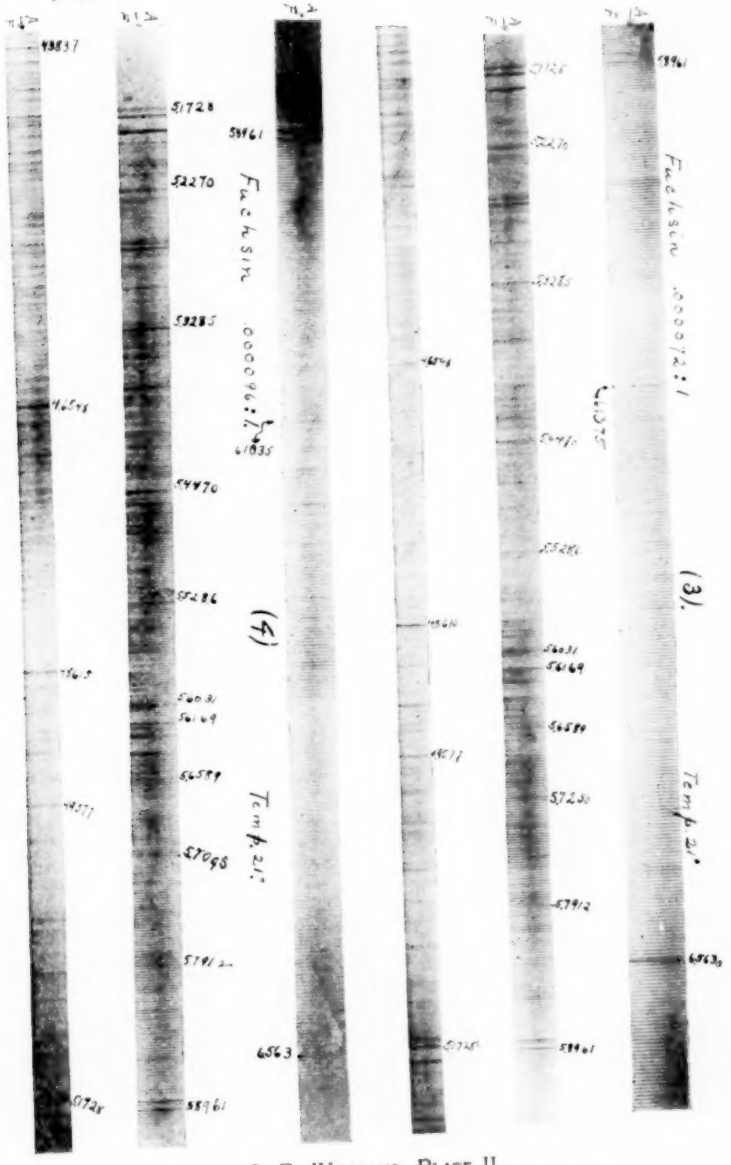
BARNARD COLLEGE,
COLUMBIA UNIVERSITY,
NEW YORK CITY.

To face page 32



S. R. WILLIAMS, PLATE I.
The Study of Dispersion by Means of Channelled Spectra.

To face page 32.



S. R. WILLIAMS, PLATE II.

The Study of Dispersion by Means of Channelled Spectra.

A STANDARD BATTERY.

By G. A. Hulett.

IN working with a potentiometer considerable inconvenience was occasioned by the variations in the batteries used. Storage batteries gave a fairly constant current for certain portions of the discharge, but during long continued observations it was necessary to frequently refer to the standard cell and make the necessary change in the regulating resistance. This inconvenience has been obviated by the use of some batteries which have given an exceedingly constant current and have practically no temperature coefficient. Since these batteries may be of use in many lines of work, where a small but steady current is needed, it has seemed well to describe them.

In 1903 Mr. Edward Weston patented a combination — Cd amalgam cadmium sulphate solution Hg_2SO_4Hg — which had an exceedingly small temperature coefficient and has been known as the Weston cell. This is a secondary standard and when calibrated served admirably as a laboratory instrument for giving a definite E.M.F. which was independent of ordinary temperature variations. This unsaturated cell, or the saturated cadmium standard cell, cannot be used as a battery, due partly to the form and dimensions given to it, but more particularly to the irregular cathode reactions. Considerable evidence has been obtained by the author,[1] showing that the cathode system of these cells ($CdSO_4 \frac{8}{3} H_2O$, Hg_2SO_4, Hg solution) was in unstable equilibrium and it was concluded that the cathode was not a reversible electrode. In studying this system by the rotation experiments, it was found that the presence of a certain amount of sulphuric acid caused the system to come to equilibrium at once. It had been assumed that the neutral cadmium sulphate solution hydrolyzed the mercurous sulphate and formed a basic salt which was responsible for slow changes in the E.M.F. of the cell, it was therefore expected that the sulphuric acid would have the effect observed.

[1] Phys. Rev., 22, 321; 23, 166; 25, 16.

It seemed therefore that it was only necessary to add a certain amount of sulphuric acid to the cell to make it completely reversible, and when a special arrangement was devised by which large electrode areas were obtained it was found possible to construct batteries which were quite reproducible and especially was it found that the mercurous sulphate could be made in the battery after it was set up, that is, it could be charged and the troublesome manipulations of preparing and washing the depolarizer entirely obviated. A great many forms have been tried, but for simplicity and effectiveness the form represented by Fig. 1 has given the best results.

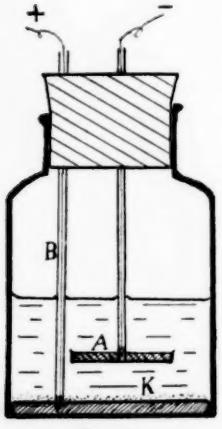


Fig. 1.

A low form salt mouth bottle about 8 cm. in diameter with a neck diameter of 5 cm., was provided with a paraffined cork. The cork was perforated by two holes and served to hold the anode cup *A* and the tube *B* through which contact with the mercury cathode *K* was made. Resting on the mercury and surrounding the anode cup was a 50 per cent. solution of cadmium sulphate to which was added 1 per cent. of sulphuric acid. By passing a current in through the mercury electrode *K*, mercurous sulphate was formed and cadmium deposited in the amalgam in the anode cup. It was found advisable to keep the solution stirred while charging.

All the materials for these batteries were at hand except the anode cups and they were not very difficult to make. A thin-walled tube, 4 cm. in diameter, was closed at one end so that the bottom was flat (Fig. 2) and in the middle on the inside of this flat bottom was sealed the little tube *C*. Before this tube was sealed on a small platinum wire was stuck to its end so that after the seal was made the platinum wire extended from the inside of the tube out into the cup in order to make contact with the amalgam and a little mercury which was placed inside the tube for contact with the lead

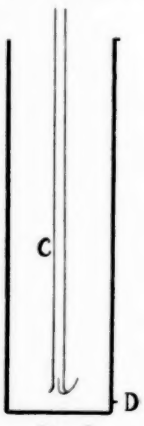


Fig. 2.

wire. The large tube was now cut off at *D*, 3 or 4 mm. above the end, with a cracking coal. (Common watch glasses were found to be of the right shape and size for the anode cups, but in fusing on the glass tube for a holder it was found that these watch glasses were not soda glass. Recourse was had to fusing two of these glasses together at the ground edge and drawing them out into tubing which was used to fuse into other watch glasses and thus very serviceable cups were made.) These cups were filled with a 12½ per cent. amalgam and the stem fastened into the cork with wax so that the bottom of the cup was about 2 cm. above the bottom of the bottle when the cork was in place. With this arrangement the electrodes were close together and the cadmium sulphate formed at the anode readily diffused over into the solution and thus concentration changes were minimized.

The bottom of the bottle was covered with mercury to a depth of ½ cm. and on this was placed the electrolyte to a depth of 4 or 5 cm. This electrolyte was made by adding 10 c.c. of strong sulphuric acid (sp. gr. 1.84) to a liter of water and in this was dissolved an equal weight of cadmium sulphate crystals (ordinary c.p. $CdSO_4 \frac{8}{3}H_2O$). For example 200 grams of the crystals were powdered and dissolved in 200 c.c. of the dilute sulphuric acid solution which made 250 c.c. of the electrolyte, just enough for one battery. The rate of solution was very slow and a motor driven stirrer was generally used to bring the salt into solution. In forming the mercurous sulphate electrolytically in the battery there is some danger of forming a small amount of a basic salt at the mercury electrode unless the solution is stirred, the stirring was easily accomplished by giving the battery a rotary motion with the hand and then a current of one ampere to 50 cm.^{2} mercury surface was used and in 20 minutes sufficient mercurous sulphate had formed. Due to the rapid deposition of cadmium there was a tendency for the amalgam to grow over the edge of the anode cup during the charging, this growth was carefully brought back into the cup. In some cases the charging was done before the anode cup was adjusted, a platinum spiral was hung in the electrolyte and a wire, protected from the electrolyte, made contact with the mercury, and then a motor-driven stirrer was used while the mer-

curous sulphate was being formed. These batteries have been charged from time to time as the surface of the mercury became exposed at any point, also occasional stirring kept the mercurous sulphate uniformly distributed over the mercury.

The first batteries were made in April, 1906, the electrolyte was *neutral* cadmium sulphate and the mercurous sulphate was made and washed in the usual way. Two of these batteries were used in series on the potentiometer, but in a few months they began to show a rapidly decreasing E.M.F. and could no longer be used, although there was still plenty of mercurous sulphate on the mercury cathodes. The electrolyte was then made acid, with sulphuric acid, and the batteries soon regained their normal value and have now been in use on the potentiometer circuit for about two years and have required very little attention. They are compact, clean and easily insulated, so are particularly convenient for the potentiometer installation suggested by Walter P. White.[1]

The first attempt to make the mercurous sulphate in the batteries was in the case of batteries Nos. III., IV. and V., which were made in small bottles with a cathode area of 20 cm.^{2}, No. III. was charged with a current of .185 amp. for 20 minutes and shortly after breaking the circuit it showed 1.02205 volts.

In 30 minutes	1.02222 volts.
2 hours	1.02226 "
6 hours	1.02214 "
2 days	1.02212 "
3 days	1.02215 "

These observations were taken as the battery stood in the laboratory subjected to ordinary temperature changes. On closing this battery over a resistance of 100,000 ohms, it immediately took up the value of 1.02200 and remained perfectly constant for 20 minutes when the resistance was lowered to 50,000 ohms and then the voltage at the battery terminals very soon dropped to 1.02187, over 40,000 ohms the voltage was 1.02180, over 30,000 1.02170, over 20,000 1.02151 and over 10,000 ohms it showed 1.02100 volts. When left on this last resistance for an hour the voltage showed a change of only 4 in the fifth place decimal as it slowly

[1] PHYS. REV., 25, 342.

decreased to 1.02096 volts. On increasing the resistance, reversing the order just given, the voltages assumed their steady values for each resistance even more rapidly than on decreasing the resistances and there was very little hysteresis as the constant values observed were only slightly lower than those just given. The internal resistance of this battery was 6.2 ohms, so there was some slight polarization. For accurate and long continued work it has been found most satisfactory to use 100 cm.^{2} cathode surface to 10,000 ohms; of course very much less resistance may be used, but then a very slow drift is noticed. No harm was done one of these batteries by directly short-circuiting the terminals for some time, for it soon regained its E.M.F. when the circuit was broken, and had only lost some mercurous sulphate which was easily restored.

Some very large batteries of this type have been made by Mr. B. J. Spence for the purpose of obtaining a steady current for a bolometer in infra-red radiation measurements. Large battery jars which allowed a mercury surface of 340 cm.^{2} were used. The wooden cover of the jar supported the four anode cups, each of 5 cm. diameter. One and a half liters of the acid cadmium sulphate solution and about 150 gr. of mercurous sulphate were put into each of these batteries, later they have been charged at a low current density without stirring. The cover of these batteries should also be provided with an "L" shaped stirrer so arranged that the arm would pass close to the surface of the mercury and with stirring the charging could be done in a short time with no danger of forming the basic salt. The first battery made by Mr. Spence showed a voltage of 1.02206, wtth an internal resistance of less than one ohm, when closed over resistances the electrode potential

dropped to	1.01691	for 100	ohms.
	1.01535	" 90	"
	1.01451	" 80	"
	1.01196	" 60	"
	1.00690	" 40	"
	1.99196	" 20	"

When left on the 20 ohms for one half an hour the p.d. was .99190. Four of these large batteries have been made and, joined in parallel have been furnishing a current of .150 amp. for bolometer work and have shown a constancy that was entirely unattainable with storage batteries.

The following observations have been made on three of the small batteries:

	No. III.	No. IV.	No. V.
February 12	1.02194	1.02190	
" 16	1.02187	1.02177	1.02198
" 23	1.02180	1.02150	1.02148
March 6	1.02155	1.02118	1.02122

During this period these batteries had been used for various purposes, where a small current was needed, and they were subjected to ordinary temperature variations, but had not been charged except when made. It would seem from the experience so far gained that the value of these batteries was the same whether the mercurous sulphate was made and washed in the usual way or made electrolytically in the batteries. They are reproducible to better than one part in 1,000, regardless of ordinary temperature changes, and the E.M.F. is 1.022 volts (Clark cell at $15° = 1.4330$ v.). When calibrated they have served for exact measurements of E.M.F. or for furnishing a small current. With a bridge of known resistance and a large regulating resistance, small electromotive forces may be accurately measured by the null method and with simple apparatus.

PRINCETON UNIVERSITY,
April, 1908.

IONIZATION IN CLOSED VESSELS.

By W. W. Strong.

Introduction.

Geitel [1] and C. T. R. Wilson [2] were the first to show that gases in closed vessels have the power to conduct electricity to a slight extent and hence are slightly ionized.

This ionization may be due to one or more of four causes, namely: (1) a spontaneous ionization of the enclosed gas itself; (2) the presence of radioactive substances in the gas or in the walls of the vessel; (3) a penetrating radiation (and secondary radiations which may be excited by this) coming from outside the vessel; (4) intrinsic radiations coming from the walls of the vessel and characteristic of the material forming the walls.

At the present time it is believed that the spontaneous ionization of the gas is very small if it exists at all. Radioactive substances have, however, been found in the air, in gases dissolved in the waters of springs, in various minerals, rocks and soils. The metals, especially lead, are known to contain minute traces of radioactive impurities. The surfaces of bodies exposed to the air are found to gather a layer of radioactive substances upon their surfaces. A penetrating radiation [3] resembling the γ radiation has been discovered. The work of Campbell,[4] Wood and others indicate that each of the various metals emits radiations peculiar to itself.

When this investigation was commenced it was considered by Eve [5] and others that it was probable that a large part of the penetrating radiation was due to radioactive products in the ground, and was constant in amount. If then any cause produced a change in the ionization within a closed vessel, this change should not be diffi-

[1] Phys. Zeit., 2, 116, 1900.
[2] Proc. Camb. Phil. Soc., II., 52, 1900.
[3] McClennan and Burton, Phys. Rev., No. 3, 1903. Cooke, Phil. Mag., p. 403, 1903.
[4] Phil. Mag., p. 206, 1906.
[5] Phil. Mag., p. 189, 1906.

cult to detect. It was thought that if an electroscope was lined with iron (French rolled steel was used in one case) and placed in a magnetic field, then this magnetic field might induce powerful enough fields about the iron atoms to modify their natural radioactivity if they possessed any. Also if the enclosed gas was magnetic, the magnetic field would slightly change the amount of energy necessary to ionize the molecules.[1] It was found that the ionization in an electroscope as measured by the rate of leak was less in a magnetic field and that this effect was larger when the field was first applied. But it was found that the variation of the rate of leak of the electroscope was so large as to make the effect of the magnetic field questionable. In order to continue the work on the effect of a magnetic field it became necessary to study the variations of the ionization in closed vessels, its cause and how it can be kept constant.

I. DESCRIPTION OF THE ELECTROSCOPES.

One type of electroscope that the writer is using is the Wilson form described in Rutherford's Radioactivity (p. 86). The other kind consists of a charged wire bent into the form of an arc and along which an earthed gold leaf moves as the potential of the wire is changed. This form of electroscope will be described shortly.

Most of the Wilson electroscopes were made of glass vessels 12.5 cm. high and 10 cm. in diameter, having a volume of one liter. These were washed with water and acids and lined either upon the inside or outside with metal foil. One that has been used a great deal was lined with aluminium foil upon the inside. Windows were made in the aluminium and circular scales were fastened upon the inside, and upon opposite sides of the gold leaf. By reading the same number on each scale, parallax was avoided. Sulphur was used as an insulator in all cases. Before putting sulphur upon a metallic surface, the metal was coated with pieces of hard rubber. These pieces of hard rubber were stuck on while they were hot and adhered to the metal very well. The sulphur was then flowed about the pieces of hard rubber.

The gold leaf was cut with the sharp edge of a piece of cane, the

[1] Langevin, Jour. de Phys., Oct., 1905.

leaf having previously been spread upon a soft, smooth cushion of chamois leather kindly given me by a gold-beater. This method is a quick and easy one. Strips 3 or 4 cm. long and .2 or .3 mm. wide can easily be cut. By pressing the ends together, strips as long as one wishes can be made.

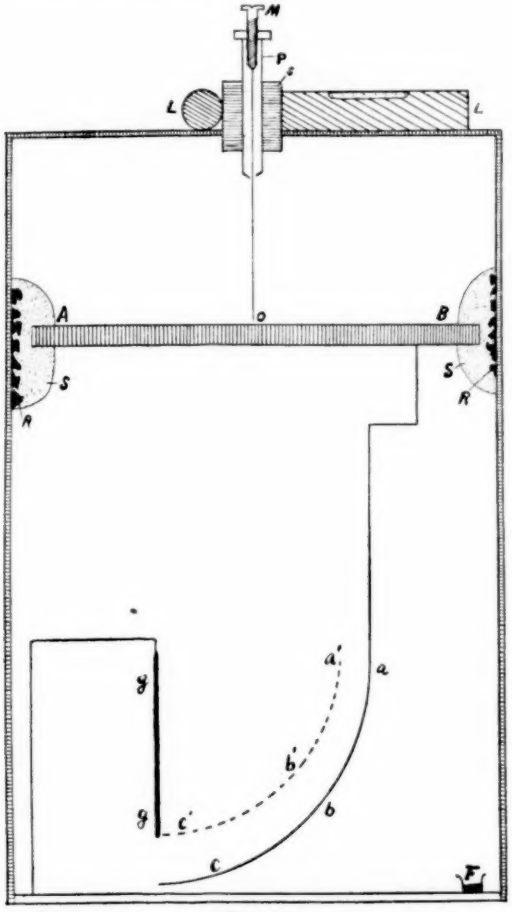


Fig. 1.

The accompanying figure shows the cross-section of a curved electrode electroscope briefly described in Science, p. 522, April 5, 1907. The dimensions of the electroscope are 12.9 × 20.1 × 10.1 cm. *R* shows some rough pieces of hard rubber upon which the sulphur *S* was flowed. *AB* is a soft iron bar, and *abc* is a copper wire.

AB is 11.7 cm. long and .528 cm. in diameter; *abc* was 16 cm. long and .060 cm. in diameter. The gold leaf was 6 cm. long, and is represented by *gg*. The inside of the electroscope is lined with sheet iron, the surface of which was polished with emery cloth.

AB is charged by means of a screw *M* and wire *MO*. A small spark gap at *O* can be made larger or smaller according as *M* is screwed up or down. When the spark gap is of convenient length sealing wax can be flowed about *M* and the electroscope will then be air-tight. *P* is a glass tube and *Q* a rubber plug. *LL* are spirit levels. The gold leaf *gg* is always earthed. As the potential of *AB abc* is raised, the gold leaf rises along the path *c'b'a'*. By properly curving *cba*, the electroscope may be made very sensitive. There are two small windows to the electroscope and *cba* is placed close enough to the one window so that the gold leaf will be in the focus of the reading micrometer microscope. Upon the window are placed threads so that the various parts of the path of the gold leaf along *c'b'a'* are marked. In most of the work a path of about .5 cm. length was used. For this length of path the motion of the gold leaf was almost linear so that the micrometer eyepiece thread when once set parallel to the edge of the gold leaf, remained so over a considerable portion of the path. Readings were taken by making the eyepiece thread tangent to a given projection on the edge of the gold leaf.

The advantages of the curved electrode electroscope are: (1) the gold leaf always turns so that its plane is parallel to the plane of the curved electrode. As the curved electrode is made parallel to the window, all parts of the gold leaf will be in the perfect focus of the reading microscope at the same time if this is perpendicular to the window; (2) as the gold leaf is earthed and the curved electrode may be made firm, the gold leaf may be placed as near the window as one likes and short focus reading microscopes may be used; (3) the electroscope is always horizontal and may easily be set up; (4) the sensibility may be made very large if the curved electrode is properly curved and has a small capacity; (5) the calibration curve is a straight line over a considerable portion of the path if the electrode is bent into the form of a circular arc. The disadvantages are that the instrument is a gravity instrument and the gold leaf is subject to air currents and radiometric effects.

II. Constancy of the Rate of Leak of Screened Electroscopes.

It was found during the course of the work that the rate of leak in closed vessels was much more constant when the external penetrating radiation was cut off by lead and iron screens.[1] A combination of lead and iron screens was used in order to be certain of cutting off both kinds of the radium γ radiations, as described by Kleeman.[2]

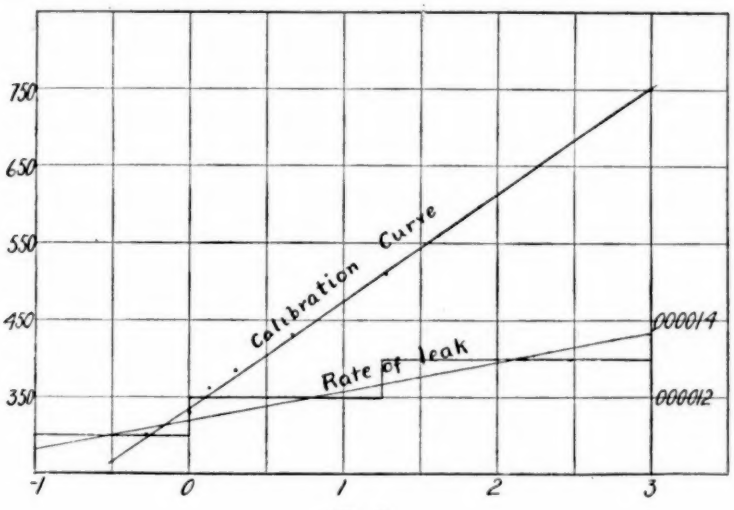


Fig. 2.

As an example showing the constancy of the rate of leak when the electroscope is screened, some readings made with the aluminium-lined liter electroscope will be given. This is one of the electroscopes previously described. Fig. 2 gives the voltage of the gold leaf for given divergences in scale divisions. It also gives the rate of leak in scale divisions per second when the electroscope was surrounded with lead and iron screens 6 to 8 cm. in thickness. This rate of leak was found to be entirely independent of external conditions and was constant for any part of the scale. The rate of leak was never found to be less than the values given in Fig. 2 under any conditions of screening. It therefore seems to be very proba-

[1] Science, July 12, 1907.
[2] Phil. Mag., Nov., 1907.

ble that this rate of leak is due to radiations from the walls of the electroscope. All other variations in the rate of leak must be due to external causes. If the rate of leak for any part of the scale under given conditions is divided by the rate of leak for the same part of the scale as given by Fig. 2, the result will give the number of times the rate of leak is greater than .000010 scale divisions per second.

The capacity of the electrode and gold leaf was approximately .85 cm. The rate of leak of the screened electroscope between the values 0 and 3 on the scale corresponds to the generation of 7 ions per cu. cm. per second using the value of $6.5\ (10)^{-10}$ electrostatic units for the charge on an ion. One can thus reduce all readings to ions generated per cu. cm. per second. In the fourth column of the following tables, .000010 is equivalent to the current that would be carried when 7 ions are formed per c.c. per second, there being a saturation current.

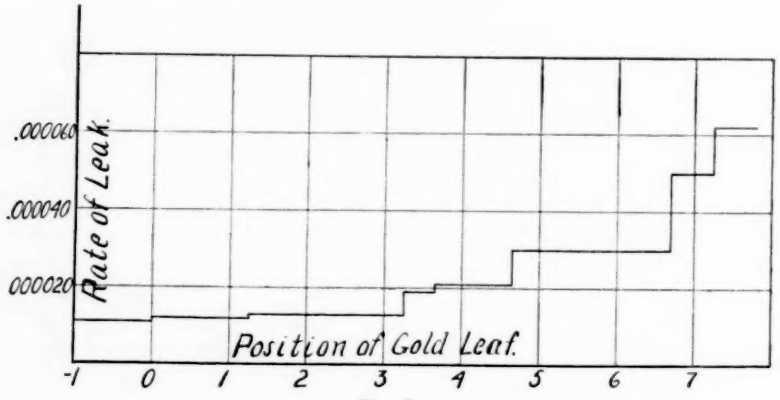


Fig. 3.

Fig. 3 was taken January, 1908. The rate of leak was also found in a cistern of rain water for the same electroscope. The cistern was dug in the ground and was large enough so that there was at least four feet of water in all directions about the electroscope. The electroscope was suspended vertically all the time. In the readings, the value .000010 for the rate of leak in scale divisions per second indicates that all the penetrating radiation has been screened off. With the exception of the reading made on Sep-

tember 11, it is to be noted that these readings are almost exactly the same as those made with the electroscope enclosed in lead screens.

TABLE I.

Electroscope in Cistern.

I.	II.	III.	IV.
Local Mean Time.	Position of Gold Leaf.	Leak in Scale Divisions Per Second.	III. Divided by the Sensibility.
Sept. 10, 1907.			
10 hrs. 4′ A. M.	7.15	.0	
3 " 41′ P. M.	6.20	.000035	.0000103
Sept. 11, 1907.			
6 hrs. 47′ A. M.	4.57	.000037	.0000130
1 " 5′ P. M.	4.07	.000022	.0000105
6 " 15′ P. M.	3.65	.000029	.0000107
Sept. 12, 1907.			
7 hrs. 1′ A. M.	2.78	.000019	.0000118
Sept. 13, 1907.			
9 hrs. 2′ A. M.	1.65	.000012	.0000098
5 " 28′ P. M.	1.25	.000012	.0000101
Sept. 14, 1907.			
8 hrs. 36′ A. M.	.60	.000012	.0000105
		Average.....	.0000106

During the night of September 10 there was a heavy thunderstorm and a considerable amount of fresh rain water ran into the cistern. As rain is known to carry down radioactive products mostly derived from radium, it is quite likely that these products caused the increased rate of leak. On account of the short period of these products their effect soon vanishes. It is interesting to note that this method might give an approximate result for the amount of radium in the air. Knowing the height from which the rain fell, and supposing all the active deposit to be carried down, one could calculate the volume of air from which the active deposit was formed. From the increased rate of leak one could calculate the amount of radium products in the rain water and all the data necessary would be known.

III. VARIATION OF THE RATE OF LEAK OF UNSCREENED ELECTROSCOPES.

When electroscopes are not screened the rate of leak is subject to wide variations. For small metallic electroscopes these vari-

ations are not large, but for large electroscopes the variations are much greater. One of the most marked examples of these variations will now be given.

During the summer of 1907 the aluminium-lined electroscope was used at the writer's home in the country one mile southeast of Mechanicsburg, Pa. The ground is almost entirely under cultivation and is of Trenton limestone formation similar to that analyzed by Eve.[1] The electroscope was placed near a large window, the curtain of which was left open so as to make the room light enough to read the electroscope. The room was kept closed so that the temperature did not vary more than 1° or 2° C. during the day. Following are some typical examples.

These examples were all taken at the same place. It is seen that the rate of leak becomes very large during the middle part of the day reaching a maximum at 11 A. M. This change of the rate of leak is either due to an increase of conduction over the sulphur insulation or to an increase of ionization due to an external radiation. As the temperature of the room did not vary greatly and as it was also kept rather dark one would not see why the insulation should be decreased during the day. Only diffused daylight fell upon the aluminium electroscope. It is quite probable, however, that the rate of leak does not vary directly as the penetrating radiation. Neither is the large value of the rate of leak during the middle of the day due to a penetrating γ-like radiation entirely, as will be shown later.

An electroscope of the curved electrode form was made of glass walls wrapped with fine copper wire. The copper wire was earthed. The rate of leak was found for this electroscope when placed in a darkened room and also when a beam of sunlight was sent through the vessel. Very large movements of the gold leaf were produced of course by the sunlight but the readings were taken an hour or more after the sunlight was removed. The sunlight did not greatly increase the rate of leak except when it was directed upon the sulphur insulation. A more careful investigation of this is to be made.

The enormous daily variation shown above may bear some relation to the increased ionization found in the open air during the day in

[1] Phil. Mag., Aug., 1907.

TABLE II.

Electroscope in the Country.

I.	II.	III.	IV.
Local Mean Time.	**Position of Gold Leaf.**	**Leak in Scale Divisions per Second.**	**III. Divided by the Sensibility.**
June 29, 1907.			
3 hrs. 29′ P. M.	4.85	.000104	.000045
5 " 15′ P. M.	4.19	.000025	.000012
7 " 23′ P. M.	4.00		
June 30, 1907.			
6 hrs. 49′ A. M.	2.98	.000025	.000012
10 " 26′ A. M.	1.59	.000107	.000082
1 " 31′ P. M.	.12	.000132	.000100
3 " 46′ P. M.	−.57	.000085	.000077
July 30, 1907.			
5 hrs. 48′ P. M.	2.90		
July 31, 1907.			
6 hrs. 6′ A. M.	.87	.000045	.000034
12 " 47′ P. M.	4.05		
5 " 27′ P. M.	.10	.000253	.000169
7 " 13′ P. M.	−.20	.000047	.000043
7 " 15′ P. M.	6.65		
Aug. 1, 1907.			
7 hrs. 12′ A. M.	3.80	.000066	.000024
Aug. 2, 1907.			
7 hrs. 5′ P. M.	4.77		
Aug. 3, 1907.			
7 hrs. 28′ A. M.	.54	.000095	.000060
6 " 20′ P. M.	5.40		
Aug. 4, 1907.			
7 hrs. 28′ A. M.	.20	.000110	.000066
10 " 51′ A. M.	3.90		
1 " 41′ P. M.	−1.00	.000480	.000359
1 " 42′ P. M.	4.40		
7 " 57′ P. M.	−.60	.000222	.000154
7 " 58′ P. M.	5.40		
Aug. 5, 1907.			
7 hrs. 50′ A. M.	.95	.000104	.000055
Sept. 2, 1907.			
2 hrs. 4′ P. M.	3.97		
5 " 49′ P. M.	1.97	.000148	.000095
8 " 53′ P. M.	1.78	.000017	.000013
Sept. 3, 1907.			
3 hrs. 6′ A. M.	1.10	.000033	.000025
7 " A. M.	.80	.000021	.000018
10 " 14′ A. M.	.14	.000057	.000050
11 " 33′ A. M.	−.57	.000171	.000155

TABLE II.—*Continued.*

Electroscope in the Country.

I.	II.	III.	IV.
Local Mean Time.	Position of Gold Leaf.	Leak in Scale Divisions per Second.	III. Divided by the Sensibility.
Sept. 3, 1907.			
12 hrs. 46′ P. M.	4.70	.000239	.000150
5 " 25′ P. M.	.70	.000050	.000042
7 " 5′ P. M.	.40		
8 " 24′ P. M.	7.70	.000057	.000010
11 " 6′ P. M.	7.15		
Sept. 4, 1907.		.000057	.000015
6 hrs. 51′ A. M.	5.56		
11 " 10′ P. M.	5.80		
Sept. 5, 1907.		.000055	.000020
6 hrs. 55′ P. M.	4.27	.000388	.000277
9 " 37′ P. M.	.50		
Dec. 22, 1907.		.000016	.000012
6 hrs. 57′ A. M.	1.25	.000024	.000020
8 " 52′ A. M.	1.08	.000061	.000050
10 " 8′ A. M.	.80	.000067	.000056
12 " 12′ P. M.	.30		
1 " 43′ P. M.	6.40	.000236	.000078
2 " 42′ P. M.	5.55	.000150	.000050
3 " 54′ P. M.	4.90	.000062	.000023
5 " 23′ P. M.	4.57	.000030	.000013
5 " 41′ P. M.	4.43	.000040	.000019
9 " 21′ P. M.	4.05		
Dec. 23, 1907.		.000023	.000011
6 hrs. 31′ A. M.	3.30		

summer as compared with the night. It may also account for the increase of conductivity of the air required by Schuster [1] to explain the changes of the earth's magnetic field as brought about by movements of bodies of charged air.

Another set of readings will be given for the electroscope placed in the tower room of the fifth floor of the physics laboratory at the Johns Hopkins University. This room contains five windows, is thus well illuminated and is below the university's telescope. Except for a thin floor and the tin roof there is nothing to cut off the penetrating radiation from above.

Table IV. gives the rate of leak when the electroscope was sur-

[1] Jour. de Phys., Dec., 1907.

rounded with a black wooden cap about .5 cm. thick, so as to prevent the entrance of any light. Table V. gives the rate of leak on the third floor of the physical laboratory, the room being surrounded by thick brick walls. Table VI. gives the rate of leak in a cave near Mechanicsburg. According to the work of Campbell and

TABLE III.

Electroscope in Tower Room.

I.	II.	III.	IV.
Local Mean Time.	Position of Gold Leaf.	Leak in Scale Divisions per Second.	III. Divided by the Sensibility.
Jan. 27, 1908.			
4 hrs. 30′ P. M.	7.35		
Jan. 28, 1908.			
8 hrs. 40′ A. M.	4.85	.000043	.000013
4 " 30′ P. M.	3.63		
Jan. 29, 1908.			
8 hrs. 55′ A. M.	2.30	.000022	.000015
Jan. 30, 1908.			
6 hrs. 5′ P. M.	−.07		
Jan. 31, 1908.			
10 hrs. 10′ A. M.	−.95	.000015	.000013
Feb. 5, 1908.			
8 hrs. 28′ A. M.	2.13		
2 " 44′ P. M.	1.74	.000021	.000016
4 " 50′ P. M.	1.60	.000017	.000013
Feb. 6, 1908.			
8 hrs. 48′ A. M.	.70	.000019	.000014
12 " 50′ P. M.	.30	.000015	.000012
4 " 40′ P. M.	.08	.000027	.000023
Feb. 7, 1908.			
10 hrs. 25′ A. M.	−.82	.000016	.000013
5 " P. M.	4.78	.000014	.000012
Feb. 8, 1908.			
8 hrs. 30′ A. M.	3.20	.000028	.000013
5 " 55′ P. M.	2.00	.000035	.000025
Feb. 10, 1908.			
10 hrs. 15′ A. M.	.58		
4 " 50′ P. M.	6.74	.000018	.000016
Feb. 11, 1889.			
8 hrs. 45′ A. M.	4.90	.000032	.000010

Wood, glass emits very little, if any, penetrating radiations. Any external radiation would therefore produce a greater relative change

TABLE IV.

Cap on Electroscope, Tower Room.

I.	II.	III.	IV.
Local Mean Time.	Position of Gold Leaf.	Leak in Scale Divisions per Second.	III. Divided by the Sensibility.
Feb. 26, 1908.			
6 hrs. 20′ P. M.	7.05		
Feb. 27, 1908.			
8 hrs. 40′ A. M.	4.93	.000041	.000012
5 " 17′ P. M.	3.96	.000031	.000013
Feb. 28, 1908.			
9 hrs. 9′ A. M.	2.74	.000021	.000012
5 " 35′ A. M.	2.13	.000019	.000015
Feb. 29, 1908.			
10 hrs. 35′ A. M.	1.18	.000015	.000012
4 " 77′ P. M.	.86	.000014	.000012
Mar. 1, 1908.			
9 hrs. 10′ A. M.	4.45		
3 " 5′ P. M.	3.92	.000022	.000011

TABLE V.

Electroscope on Third Floor.

I.	II.	III.	IV.
Local Mean Time.	Position of Gold Leaf.	Leak in Scale Divisions Per Second.	III. Divided by the Sensibility.
Jan. 13, 1908.			
11 hrs. 44′ A. M.	5.55		
1 " 29′ P. M.	5.10	.000071	.000024
2 " 55′ P. M.	4.75	.000068	.000023
5 " 2′ P. M.	4.33	.000055	.000024
Jan. 14, 1908.			
8 hrs. 36′ A. M.	2.50	.000033	.000026
10 " 12′ A. M.	2.28	.000038	.000029

in the rate of leak in a glass vessel. Several electroscopes of glass were made. All metal parts were covered with sulphur and the outside of the electroscope was covered with aluminium foil. The following readings are typical.

Time.	Position of Gold Leaf.	Rate of Leak.
Nov. 25, 1907.		
4 hours, 51′ P. M.	12.70	
Nov. 26.		
8 " 10′ A. M.	11.95	.000013
5 " P. M.	11.05	.000038
Nov. 27.		
8 " 10′ A. M.	10.35	.000013
4 " 42′ P. M.	9.14	.000039
Nov. 28.		
8 " 30′ A. M.	8.45	.000012

The above table shows that during the summer the rate of leak was much larger. During the nights of summer the rate of leak is very large as a rule, at least for the place observed. The readings for September 2 and 3 show a minimum at about 8 P. M. and a slight maximum about midnight.

If the penetrating radiation comes largely from the air, then the variations should be almost as great when all the electroscope is

TABLE VI.

Electroscope in Cave.

I.	II.	III.	IV.
Local Mean Time.	Position of Gold Leaf.	Leak in Scale Divisions per Second.	III. Divided by the Sensibility.
Sept. 28, 1907.			
8 hrs. 4′ A. M.	6.70		
11 " 43′ A. M.	5.90	.000061	.000017
7 " 41′ P. M.	4.66	.000043	.000015
Sept. 29, 1907.			
8 hrs. 32′ A. M.	3.70	.000021	.000010
5 " 3′ A. M.	2.90	.000029	.000016
Dec. 24, 1907.			
3 hrs. 52′ P. M.	7.00		
Dec. 25, 1907.			
8 hrs. 13′ A. M.	5.08	.000033	.000010
12 hrs. 20′ P. M.	4.64	.000030	.000011
4 " 7′ P. M.	4.30	.000025	.000011
Dec. 26, 1907.			
8 hrs. 6′ A. M.	3.05	.000022	.000011
12 " 30′ P. M.	2.78	.000019	.000014
5 " 10′ P. M.	2.47	.000018	.000013
Dec. 27, 1907.			
9 hrs. 15′ A. M.	1.44	.000018	.000013
5 " 40′ P. M.	1.00	.000014	.000012

screened except the top. This was done, the electroscope being placed on the roof of the physical laboratory.

TABLE VII.

Cap on, Electroscope on Roof.

I.	II.	III.	IV.
Local Mean Time.	Position of Gold Leaf.	Leak in Scale Divisions per Second.	III. Divided by the Sensibility.
March 10, 1908.			
5 hrs. 5′ P. M.	3.97		
March 11, 1908.			
8 hrs. 15′ A. M.	3.05	.000017	.000010
March 12, 1908.			
8 hrs. 5′ A. M.	5.70	.000047	.000018
5 " 21′ P. M.	3.83		
March 13, 1908.			
8 hrs. 40′ A. M.	2.78	.000019	.000012
6 " 14′ P. M.	2.12	.000019	.000015
March 14, 1908.			
8 hrs. 40′ A. M.	1.40	.000014	.000011

These tables show that the rate of leak for the unscreened electroscope varies between wide limits but it is never less than the rate of leak for the screened electroscope. The variation is much less in Baltimore than in the country. The rate of leak in a cave is much less than the rate of leak observed on the surface and the variations are very much less. If the larger part of the external radiation comes from the ground, then in a cave the rate of leak should be twice as great as on the surface of the ground, but this is not so. The rate of leak at night in the physical laboratory is fairly constant compared with the leak during the day. The readings during the night in the country are much larger during the summer. The screening of all parts of the electroscope except the top and placing the electroscope thus screened on the roof of the physical laboratory did not decrease the variations very greatly.

IV. THE DIURNAL PERIOD OF THE RATE OF LEAK.

Some of the results obtained with the unscreened "sheet-iron" electroscope will now be given. This electroscope was used in the tower room. Readings were made with a micrometer microscope,

the micrometer being in the eyepiece. Both micrometer microscope and electroscope were kept fixed, so that only a small portion of the path of the gold leaf could be used. When the gold leaf passed out of the field of the microscope, the electroscope would be recharged. In the curves these intervals are represented by dotted lines. When lead screens were placed around the electroscope, it was found that the natural ionization in the instrument produced a constant rate of leak and therefore the fluctuations observed with the unscreened electroscope as given by the following curves were due to external causes. All the curves except Fig. 9 were taken in the tower of the physical laboratory. This latter was taken near Mechanicsburg, Pa. These and other curves show maxima at 9 A. M. and 10 P. M., marked minima at 7 A. M. and 6 P. M.[1] and a less marked one about noon. Wood and Campbell[2] find maxima at 9 A. M. and 10 P. M., minima at 2 P. M. and 4 A. M. McKeon[3] finds maxima at about 10 A. M. and 11 P. M. and minima at 6 P. M. and 7 A. M.

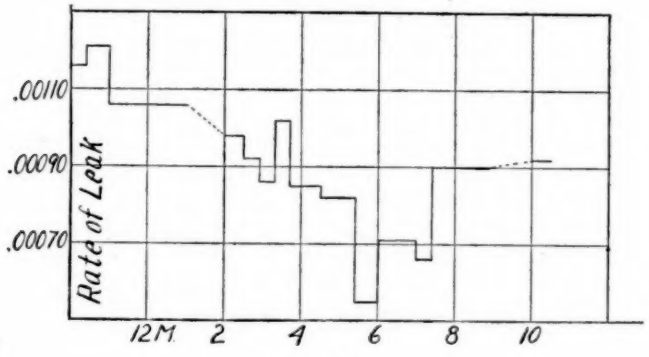


Fig. 4.

Description of Curves. — Fig. 4. Ionization curve for February 19, 1907, taken in the "tower room." Weather clear. Fig. 5. Ionization curve for February 26, 1907, taken in the "tower room." Weather clear and warm. The temperature variation in the room during the day was but 2° C. Mean barometer 76.60

[1] Science, July 12, 1907.
[2] Phil. Mag., Feb., 1907.
[3] PHYS. REV., Nov., 1907.

cm. Sleet in evening about 8 P. M. Fig. 6. Ionization curve for February 25, 1907, taken in the "tower room." Weather clear. The day previous a snow eight inches deep had fallen and this still covered the ground. If the penetration radiation had

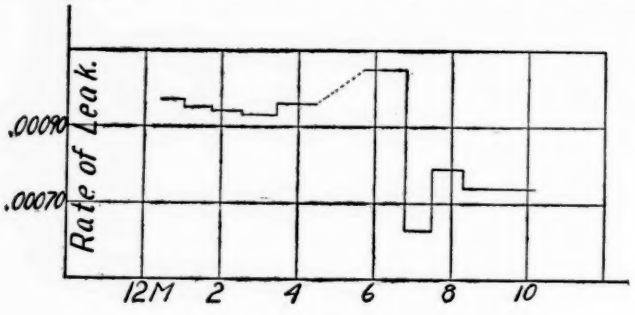


Fig. 5.

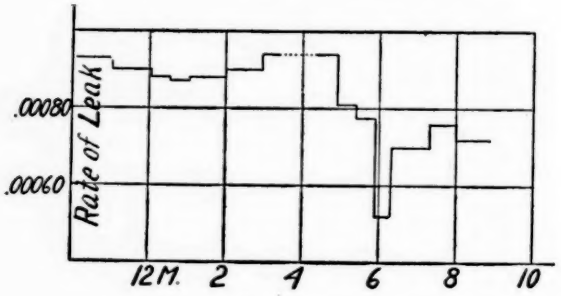


Fig. 6.

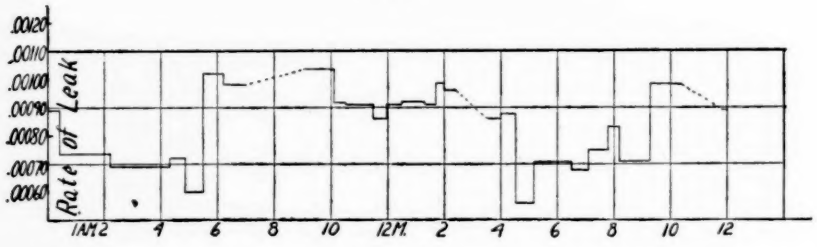


Fig. 7.

come from the ground the snow would probably have screened a considerable part of it. But this is not the case. Barometer 76.25 cm. In Fig. 5 it is seen that during the sleet the ionization is quite small. Fig. 7. Ionization curve for February 23, 24, 1907, for the "tower room." Weather clear and cold. Barometer 77.80

cm. Temperature range 11° to 15° C. Fig. 8. Ionization curve, March 9, 10, 1907, taken in the "tower room." At midnight it began to snow. The curve shows how the ionization has thereby been decreased. The barometer decreased gradually from 77.00 cm. at the beginning of the observations and dropped to 76.00 cm. at the end of the observations. The temperature varied

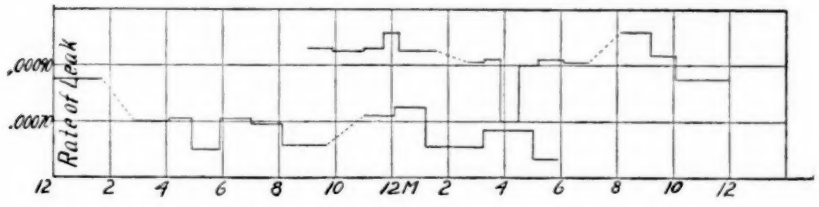


Fig. 8.

between 12° and 18° C. Fig. 9. Ionization curve taken December 26, 1907 near Mechanicsburg, Pa.

The rate of leak over a considerable part of the scale of the fixed microscope when the electroscope was screened was found for the "sheet-iron" electroscope. A calibration curve for this

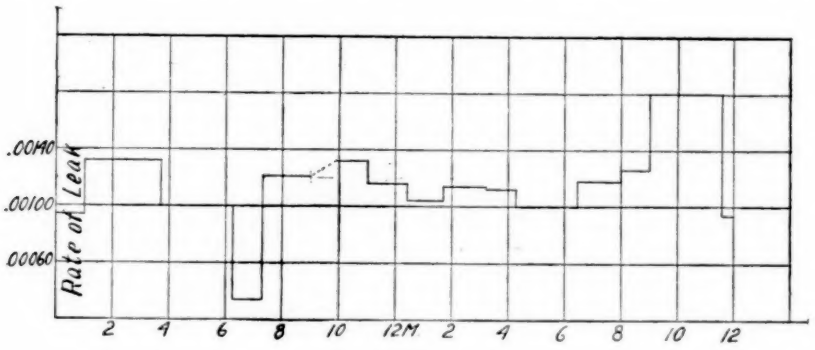


Fig. 9.

part of the scale was then drawn as for the aluminium electroscope. This calibration curve thus gave the relative sensibility of the different parts of the scale. The rate of leak over any part of the scale divided by the sensibility ought to be unity for the screened electroscope and greater than unity for the unscreened electroscope. This will be shown by the following figures.

I.	II.	III.	IV.
Local Mean Time.	Position of Gold Leaf.	Leak in Scale Divisions per Second.	III. Divided by the Sensibility.
April 8, 1907.			
8 hrs. 34′ A. M.	10.814	.000334	1.095
3 " 34′ P. M.	18.403		
8 " 55′ P. M.	6.978		
April 9, 1907.			
8 hrs. 20′ A. M.	19.890	.000318	1.092
April 10, 1907.			
10 hrs. 53′ A. M.	11.896	.000324	1.062
5 " 44′ P. M.	19.900		
May 1, 1907.			
11 hrs. 56′ A. M.	.117	.000273	1.066
10 " 58′ P. M.	10.278		
May 2, 1907.			
11 hrs. 55′ A. M.	.112	.000301	1.153
9 " 43′ P. M.	11.483		

The average for twenty one days for column IV. was 1.147. Now to show the great variation in the rate of leak the top part of the screen was removed. The following readings show that the

I.	II.	III.	IV.
Local Mean Time.	Position of Gold Leaf.	Leak in Scale Divisions per Second.	III. Divided by the Sensibility.
May 10, 1907.			
11 hrs. 57′ A. M.	1.60	.000500	2.03
3 " 14′ P. M.	7.683		
5 " 28′ P. M.	.629	.000398	1.59
10 " 53′ P. M.	8.856		
May 11, 1907.			
12 hrs.	1.889	.000690	2.65
3 " 57′ P. M.	10.070		
May 13, 1907.			
1 hrs. 46′ P. M.	−.548	.000270	1.11
10 " 15′ P. M.	7.310		
May 14, 1907.			
10 hrs. 58′ A. M.	.710	.000357	1.37
7 " 50′ P. M.	11.306		
May 16, 1907.			
12 hrs. 54′ P. M.	1.487	.000600	2.22
6 " 50′ P. M.	14.740		
May 17, 1907.			
10 hrs. 56′	−.253	.000407	1.63
5 " 45′	9.270		

external radiation entering at the top of the electroscope was what produced the great variations in the rate of leak.

V. The Rate of Leak During a Snow or Rain.

The effect of a snow in decreasing the external radiation is shown very well by Fig. 8. Here it began to snow about midnight. Upon the scale given, .00049 is the rate of leak in scale divisions per second when the electroscope is screened. In Table IV., March 1, 1908, there was rain during the day. February 5, 1908, there was a snow. September 2, 1907, there was a thunderstorm during the afternoon. During the night of September 3 there also was rain. These data show that in these cases the penetrating radiation fell almost to zero value. This is to be expected since rain and snow are known to carry down radium A, B and C. On account of the quick recovery of the external radiation it seems very probable that the emanation remains in the air.

VI. The Origin of the Penetrating Radiation.

Since the penetrating radiation resembles the γ rays from radioactive substances it is very probable that it originates from radioactive substances. The problem then is to locate these radioactive substances. From the radium content of the various rocks as determined by both Strutt and Eve it appears that there is not enough radium in the ground to account for the penetrating radiation.[1] Taking a value of the radium content which is probably large, the γ radiation which this would be the source of, would produce less than one ion per c.c. per second. Cooke found the penetrating radiation to produce 4.5 ions per c.c. per second, McClennan 9 ions per c.c. per second, and for an electroscope with a cap the writer found that the penetrating produced 4 ions per c.c. per second when screened on the under side. Firstly, then, the radium content of the ground is not sufficient to produce a sufficiently powerful penetrating radiation. Secondly, a penetrating radiation coming from radioactive products in the earth should be constant in amount. The penetrating radiation varies considerably in value. Thirdly, during a rain or snow the penetrating radiation decreases very much

[1] Terr. Mag., Dec., 1907.

in value. Rain and snow carry radioactive products into the ground and increase the radioactive content of the ground, without increasing the penetrating radiation. Fourthly, screens placed about electroscopes on all sides except the top should absorb practically all the penetrating radiation from the earth. Experimentally, however, this is not found to be the case. Fifthly, the penetrating radiation in a cave should be larger than the penetrating radiation on the surface if this has its source in the ground. The writer has, however, found the penetrating radiation less in a cave. The conclusion is that under the conditions of investigation the larger part of the penetrating radiation did not come from the ground.

On the other hand a very simple method of explaining the above results is to suppose that the penetrating radiation comes largely from radioactive products in the air. Suppose that a considerable part comes from radium C which has been broken down from the emanation. Now it is known that most of the radioactive products carried down by rain or snow consist mostly of radium products.[1] During a heavy rain or snow most of the radium products would be carried down and so the penetrating radiation would be considerably decreased.

If the emanation remains in the air, then in a few hours after the snow or rain, the products radium A, B and C will soon be in equilibrium again, and the penetrating radiation will increase to its normal value. This has been noticed to occur in about two hours in one case. This theory would explain why the active deposit occurs in rain or snow even if the storm has been a long one, the active deposit being continually formed from the radium emanation. As the radium emanation has been shown by Elster and Geitel,[2] Ebert and Ewers[3] and others to diffuse up through the ground, it is quite probable that the radium emanation in the air comes largely from the ground. The amount of emanation would then vary with the direction and strength of the wind. The air over large areas of low barometer would contain more emanation and this air would be carried about by cyclonic and anticyclonic storms.

[1] J. S. Allen, PHYS. REV., p. 306, 1903; C. T. R. Wilson, Proc. Camb. Phil. Soc., 11, p. 428, 1902; J. Kaufmann, Meteorologicale Zeit., March, 1905; Costanzo and Negro, Phys. Zeit., 7, pp. 350, 921, 1906.

[2] Phys. Zeit., 3, p. 574, 1902.

[3] Phys. Zeit., 4, p. 162, 1902.

Recent determinations by Eve [1] of the content of the air of radium emanation do not bear out the above theory quantitatively, the emanation content being too small to account for the penetrating radiation. But at the same time the penetrating radiation may have been small. Eve [2] then suggests that the penetrating radiation may come from thorium products in the ground.

As the γ radiation from thorium would require 6.9 (10^6) times [3] as much thorium in weight as radium, the amount of thorium necessary to produce the penetrating radiation would be quite large. It would be very interesting to make a thorium survey of various minerals and rocks. Recent work by Dadourian [4] does not indicate a large thorium content in the ground.

It hardly seems possible to account for the variations of the penetrating radiation if this comes from radioactive products in the ground. This difficulty is especially pronounced in a city where all the rain water runs away immediately after its fall, it not being able to wash the active deposit down into the ground. In some localities, however, there may be enough radioactive material in the ground to explain the penetrating radiation.

The diurnal variations are much more difficult to explain. If the larger part of the penetrating radiation comes from the active deposit in the air, and this is probably largely condensed on the "large" ions of Langevin, then if these large ions were suddenly carried down to the ground there would be a minimum ionization. But how this could happen is not evident. If the "large" ions are numerous and the active deposit is attached to these, then the active deposit would be more difficult to draw out of the air by means of charged wires. If the active deposit was attached to the "small" ions in the air, then it would be more easily acquired by a charged wire. This may explain some of the variations in the amount of active deposit formed on charged wires.

According to the recent work of Cameron and Ramsay, helium is formed from the radium emanation when the latter is mixed with a gas, argon and neon, when the emanation is mixed with certain

[1] Phil. Mag., Dec., 1907.
[2] Nature, p. 486, March 26, 1908.
[3] Am. Journ. of Science, Dec., 1906.
[4] Am. Journ. of Science, Apr., 1908.

liquids. It might be possible that a part of the helium and argon in the atmosphere is formed in this way. If one knew the amount of radium emanation in the air, a calculation of the amount of helium formed could be made. If, then, helium is not accumulating in the atmosphere, this means that the amount of helium formed is equal to the amount which the earth's atmosphere loses to space.

VIII. CONCLUSION.

1. A form of electroscope with a curved electrode and earthed gold leaf is very convenient in working with comparatively large vessels.

2. The ionization in closed vessels investigated has been found to be constant when the vessels are surrounded with lead and iron screens. The rate of leak due to this ionization has been used to calibrate the electroscope.

3. The variation of the rate of leak of an unscreened electroscope is due to external radiations. Part of this external radiation is the so-called penetrating radiation.

4. The external radiation is greater during the summer.

5. The curved electrode electroscope has a double diurnal period in its rate of leak.

6. During a rain or snow the penetrating radiation decreases very considerably for the cases observed. In several instances the penetrating radiation has reached almost a zero value.

7. An electroscope open at the top but with thick screens on all sides and on the bottom, is subject to very considerable variations in its rate of leak. From this and other reasons it seems very probable that a considerable part of the penetrating radiation comes from radioactive products in the air.

8. Since the coefficient of absorption for air is small, the penetrating radiation from radioactive products in the air comes from large bodies of air at considerable distance from the place of observation. A study of this penetrating radiation at different places would indicate the origin and movements of radioactive matter in the air.

The writer's thanks are extended to Professor Ames for his help and kindness throughout the course of the investigation.

JOHNS HOPKINS UNIVERSITY,
April, 1908.

A GENERAL FORMULA FOR DISPERSIVE AND RESOLVING POWERS.

By L. P. Sieg.

THE derivation of expressions for the resolving powers of prisms and diffraction gratings, as found in most works on optics, usually follow in a general way Rayleigh's [1] method, while Michelson's [2] proof is usually used for the echelon grating. It was suggested to the writer some time ago by C. V. Kent, of Carleton College, that a single general formula for the resolving power of any instrument of this kind might be developed which would include each of the above three as a special case. The following treatment which leads to such a formula will possibly be of interest to teachers of optics, not only on account of its comparative simplicity, but also because it shows how closely connected these three forms of spectroscopic apparatus are. As far as the writer knows no such general formula has previously been published.

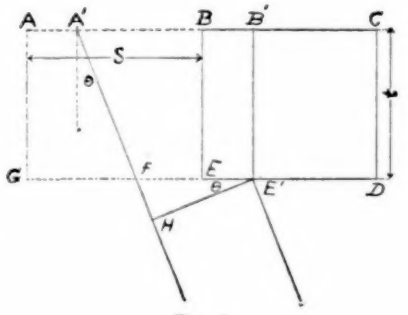


Fig. 1.

In Fig. 1, let ABC be the intersection with the plane of the paper, of a portion of a plane wave front. Let $BCDE$ be the base of a rectangular prism of a transparent medium of index μ_1. The altitude of the prism is of no concern. Let this prism be immersed in a medium of index μ_2, of which a like prism of base $ABEG$ is considered. The intersections of the front faces of these prisms with the plane of the paper are BC and AB, of equal length s. Let the depth of the prisms be t. Take A' and B' as corresponding

[1] "Wave-Theory," Ency. Brit., Vol. XXIV. "Scientific Papers," III., 47.
[2] "The Echelon Spectroscope," Astrophys. Jour., 8, p. 36, 1898.

points of the sources, *i. e.*, AA' equals BB'. Then $A'B'$ is equal to s. Consider the portion of any wave front HE' diffracted through such an angle θ, that the phase difference between H and E' is a multiple of 2π. In general this is not the original wave front. The geometrical path difference of the extreme portions of the wave is $\pm s\theta$, depending on the sign of θ. The physical path difference is $\mu_1 t - \mu_2(t \pm s\theta)$. Usually the second medium is air and taking the index for air as practically unity, and independent of the wave-length,

$$\mu_1 t - (t \pm s\theta) = m\lambda. \tag{1}$$

For the present taking only the negative value of θ, we have for the dispersive power, writing μ for μ_1,

$$\frac{d\theta}{d\lambda} = \frac{m}{s} - \frac{t}{s}\frac{d\mu}{d\lambda}. \tag{2}$$

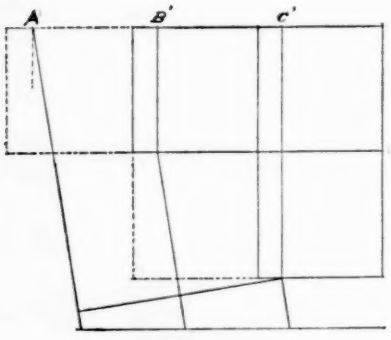


Fig. 2.

The extension of the above relation to three sources is readily seen from Fig. 2, where the dotted lines represent the necessary portions of the air medium to give the equal width of sources, and A', B' and C' are corresponding points. The validity of the proof is in no way affected by supposing equal and corresponding portions of the adjoining sources to be made opaque. For the resolving power, from equation (2),

$$\frac{\lambda}{d\lambda} = \frac{\lambda}{d\theta}\left(\frac{m}{s} - \frac{t}{s}\frac{d\mu}{d\lambda}\right). \tag{3}$$

In order that two images of a rectangular slit, separated by $d\theta$, may be resolved, $d\theta$ must never be less than λ/e, where e is the effective breadth of the beam.

At the limit we can put $d\theta$ equal to λ/e and obtain for the resolving power

$$\frac{\lambda}{d\lambda} = e\left(\frac{m}{s} - \frac{t}{s}\frac{d\mu}{d\lambda}\right). \tag{4}$$

Applying equation (4) to a grating,

$$\frac{d\mu}{d\lambda} = 0 \quad \text{and} \quad \frac{\lambda}{d\lambda} = e \cdot \frac{m}{s}.$$

Multiplying by n/n, where n represents the number of pairs of corresponding sources, AB and BC,

$$\frac{\lambda}{d\lambda} = \frac{em}{s} \cdot \frac{n}{n} = mn \tag{5}$$

since ns, the width of the grating, is equal to e. Applying (4) to the prism, $m = 0$, and

$$\frac{\lambda}{d\lambda} = -\frac{et}{s}\frac{d\mu}{d\lambda}$$

and as before multiplying by n/n,

$$\frac{\lambda}{d\lambda} = -\frac{et}{s} \cdot \frac{n}{n} \cdot \frac{d\mu}{d\lambda} = -T\frac{d\mu}{d\lambda},$$

where $e = ns$, and $T = nt$, the thickness of the prism at the base. In this case we should have used the positive value of $s\theta$, Eq. (1), so that in this case

$$\frac{\lambda}{d\lambda} = T\frac{d\mu}{d\lambda}. \tag{6}$$

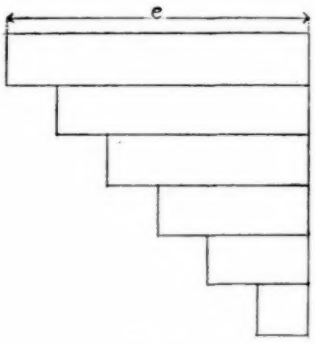


Fig. 3.

From our knowledge of $d\mu/d\lambda$ we know that the sign of the above expression is intrinsically negative. Fig. 3 shows how a pile of infinitesimally thin plates would build up a prism. It shows at once that a prism is an echelon grating with an infinite number of steps, each of infinitesimal breadth and thickness. Of course we can here use only the spectrum of zero order, and could find the spectrum of first order only at infinity. This is obvious since A and B, the corresponding points of the sources, are practically coincident, so there is no possibility of obtaining a path difference of λ in a finite field.

Applying (4) to the echelon, we have from (1),

$$m = \frac{(\mu - 1)t}{\lambda},$$

since $s\theta$ can be neglected in comparison with t,

$$\frac{\lambda}{d\lambda} = \frac{et}{\lambda s}\left(\mu - 1 - \lambda\frac{d\mu}{d\lambda}\right)\cdot\frac{n}{n}$$

$$= \frac{nt}{\lambda}\left(\mu - 1 - \lambda\frac{d\mu}{\lambda d}\right), \qquad (7)$$

since $ns = e$.

We may readily write the expressions for the dispersive powers, obtaining them from (2) in a manner similar to the foregoing. For the grating

$$\frac{d\theta}{d\lambda} = \frac{m}{s}\cdot\frac{n}{n} = mn' \qquad (8)$$

where $n' = n/s$, the number of lines per unit length.

For the prism, changing the sign of the last term, as before

$$\frac{d\theta}{d\lambda} = \frac{t}{s}\frac{d\mu}{d\lambda}\left(\frac{n}{n}\right) = \frac{T}{e}\frac{d\mu}{d\lambda}, \qquad (9)$$

an intrinsically negative quantity. Here ns equals e, the altitude of the base triangle of the prism.

For the echelon it is readily seen that

$$\frac{d\theta}{d\lambda} = \frac{t}{s\lambda}\left(\mu - 1 - \lambda\frac{d\mu}{d\lambda}\right), \qquad (10)$$

or with everything else constant, the dispersive power depends on the ratio of the thickness of a plate to the breadth of a step.

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April 15, 1908.